Local symplectic invariants and generalized Darboux Theorem

S. Janeczko

If L is a subspace of a symplectic space (X, ω), then obviously the form $\omega\mid_L$ and its exterior powers ω^k (volume form in particular) are symplectic invariants. The natural questions are raised by this property: Does the assumption that such linear subspace data are preserved determine interesting transformations of the symplectic vector space? And is it possible to collect such partial "information" from a finite family of subspaces and construct a complete system of invariants for the symplectic group? To answer these questions we establish the structural data on subspaces and find the groups of automorphisms defined by them. We discuss the case of a finite family of 2k-dimensional subspaces of a symplectic vector space and describe the properties of linear automorphisms preserving the symplectic data ω^k. The crucial property that the subspaces of the system are not co-planar, that is, they do not belong to any hyperplane in the appropriate Grassmannian, allows us to get symplectomorphisms or even more generally linear symplectic relations or conformal symplectomorphisms. In the case of singular $L \in R^{2n}$ we prove the genralized version of Darboux-Givental theorem. If ω_1, ω_2 are symplectic forms on R^{2n} with the same algebraic restriction to L, i.e. $\omega_1 - \omega_2 = \alpha + d\beta$ where α and β are vanishing on L ($\alpha(x) = 0, \beta(x) = 0$ for all $x \in L$) and $\omega_1 - \omega_2$ is trivial in $H^2(L)$ then there exists a local diffeomorphism germ which transforms ω_1 into ω_2. In the case of curves, we derive their symplectic classification and describe the basic symplectic invariants like symplectic defect and symplectic codimension being recognized as a Delta invariant.