

MANIFOLDS WITH A UNIQUE EMBEDDING

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ABSTRACT. Let X be a smooth, compact manifold of dimension k . We show that any two smooth embeddings $X \rightarrow \mathbb{R}^n$ are positively tamely equivalent provided $n \geq 2k + 2$. Moreover, if additionally X is a real analytic manifold, then we show (which seems to be a quite new result) that any two real analytic embeddings $X \rightarrow \mathbb{R}^n$ are real analytically equivalent.

We extend this result to some interesting sub-categories of the category of real smooth manifold. In particular we will prove that if X, Y are Nash k -dimensional submanifolds of \mathbb{R}^n (where $n \geq 2k + 2$) and $\phi : X \rightarrow Y$ is a diffeomorphism (Nash isomorphism, real analytic isomorphism), then ϕ can be extended to a positively tame diffeomorphism (Nash isomorphism, real analytic isomorphism) $\Phi : \mathbb{R}^n \rightarrow \mathbb{R}^n$.

We prove also that if X, Y are k -dimensional smooth algebraic subvarieties of \mathbb{C}^n (where $n \geq 2k + 2$), and $\phi : X \rightarrow Y$ is a biholomorphism (or polynomial isomorphism), then ϕ can be extended to a global tame biholomorphism (polynomial automorphism) $\Phi : \mathbb{C}^n \rightarrow \mathbb{C}^n$.

Finally we prove: if X, Y are k -dimensional closed semi-algebraic subsets of \mathbb{R}^n (where $n \geq 2k + 2$), and $\phi : X \rightarrow Y$ is a semi-algebraic homeomorphism, that ϕ can be extended to a global positively tame semi-algebraic homeomorphism $\Phi : \mathbb{R}^n \rightarrow \mathbb{R}^n$. This last result is a semi-algebraic counterpart of a classical result of Herman Gluck, on extension of homeomorphisms of compact polyhedrons.

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