Shell Exploration & Production

Algebraic Oil

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Overview

Traditional Research

- Exploration: Geophysics
- Production: Reservoir Engineering

A First Turning Point

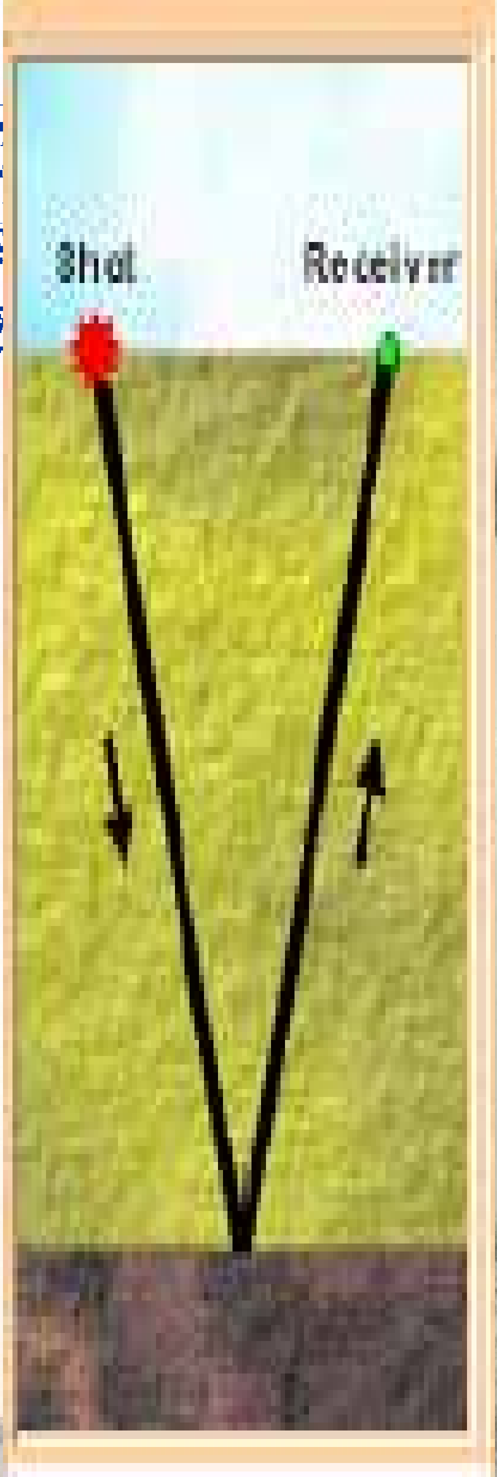
- Production Operations

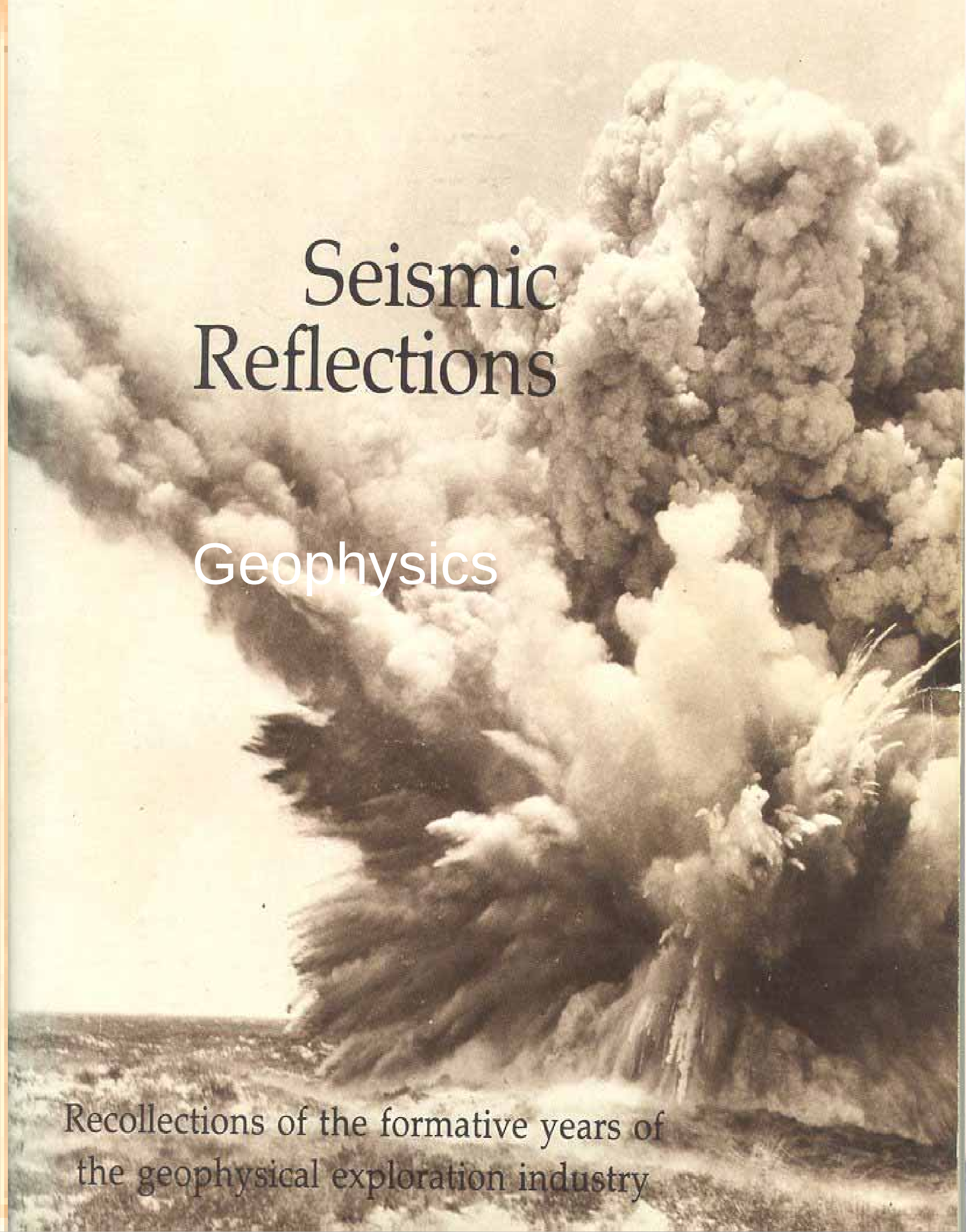
A Second Turning Point

- Production Operations Revisited
  - The Decomposition Problem (D. Heldt, A. Kehrein, S. Pokutta)

The Dortmund – Shell connection

Geophysics





Seismic Reflections

Recollections of the formative years of the geophysical exploration industry

A refraction shot in West Texas. 2,000 pounds of dynamite shot by Humble Oil & Refining Company (now Exxon), 1930.

From: Petroleum Handbook (Shell, 1948)

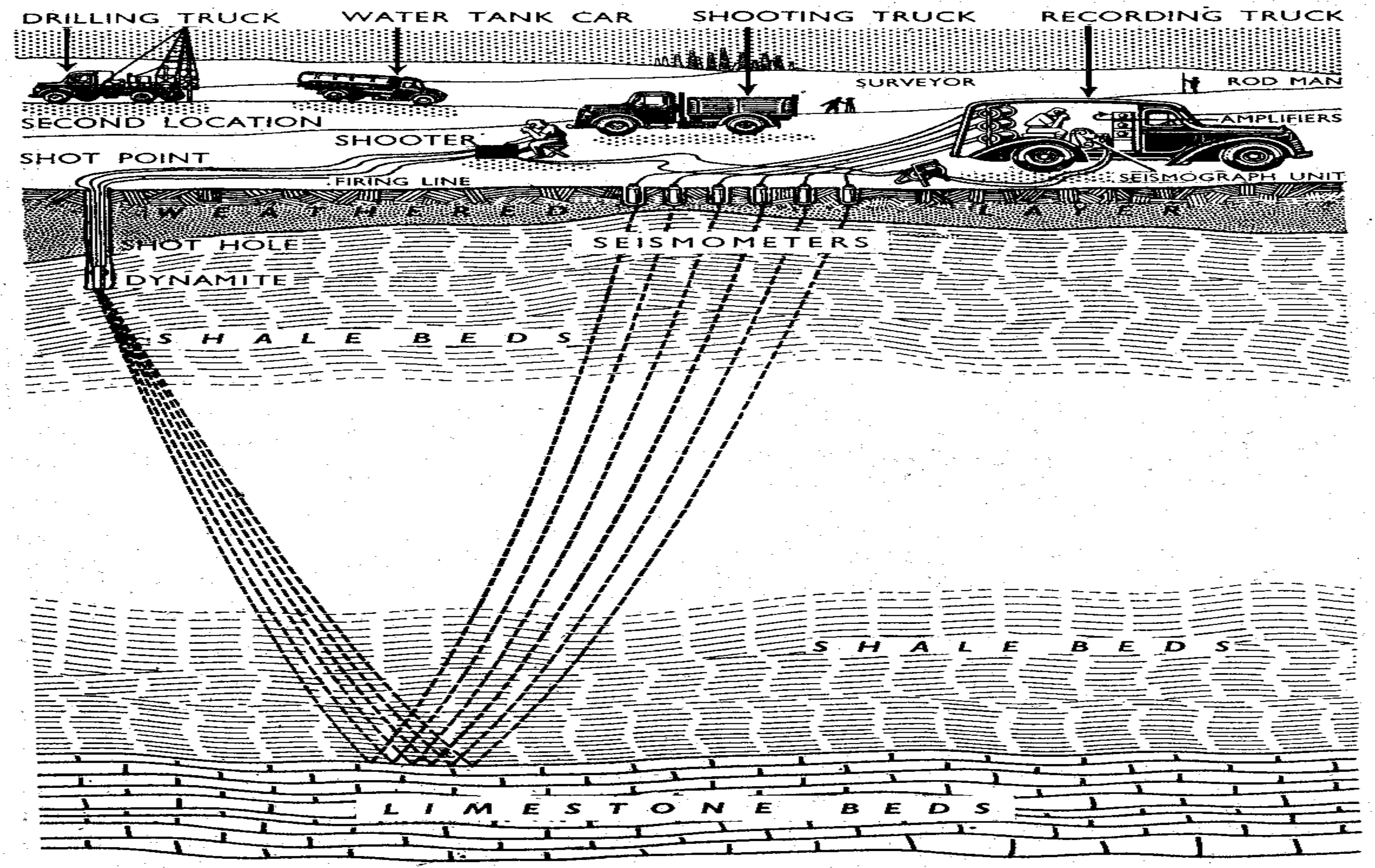
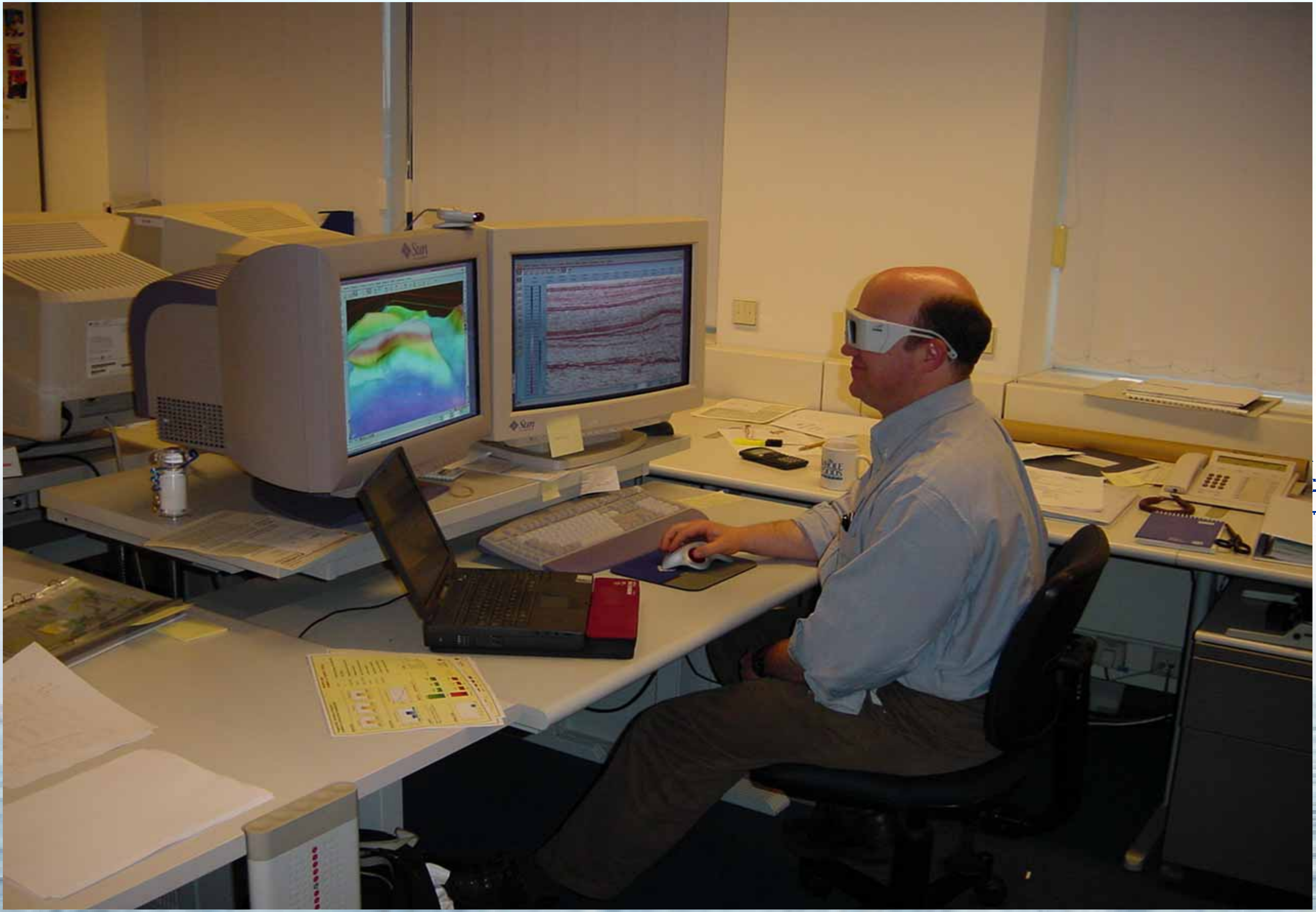


Fig. 14. Diagrammatic scheme for Reflection Shooting.

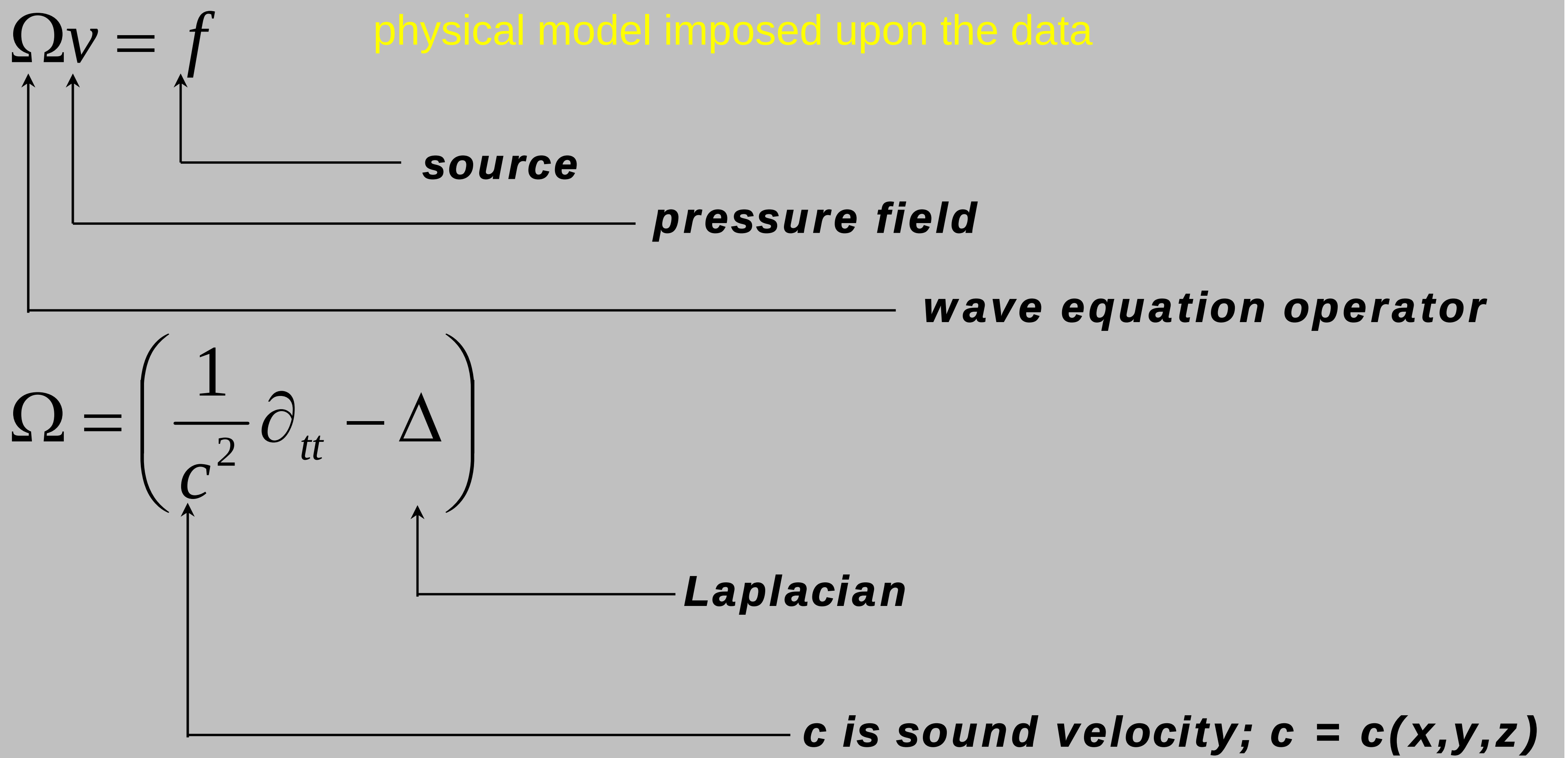
Modern technology



(Jaap van der Toorn, NAM TGS-S)

The Interpreter at work

wave equation



$$\Omega v = f$$

physical model imposed upon the data

source

pressure field

wave equation operator

$$\Omega = \left(\frac{1}{c^2} \partial_{tt} - \Delta \right)$$

Laplacian

c is sound velocity; c = c(x,y,z)

specification *c* for different layers is velocity model *m*

$$\Omega = \Omega(m)$$

Inversion

Find the best model *m* that explains the data

$$\min_{m} \; J(m) \qquad \text{with}$$

$$J(m) = \sum_{l} (v_l(m) - v_l^{\text{obs}})^2$$

$$s.t., \Omega(m)v = f$$

General approach too difficult, *J* has local minima

Migration: an initial model, m_0, is assumed known

A large number of approaches for the Migration problem
Here just one example

wave equation

Helmholtz equation

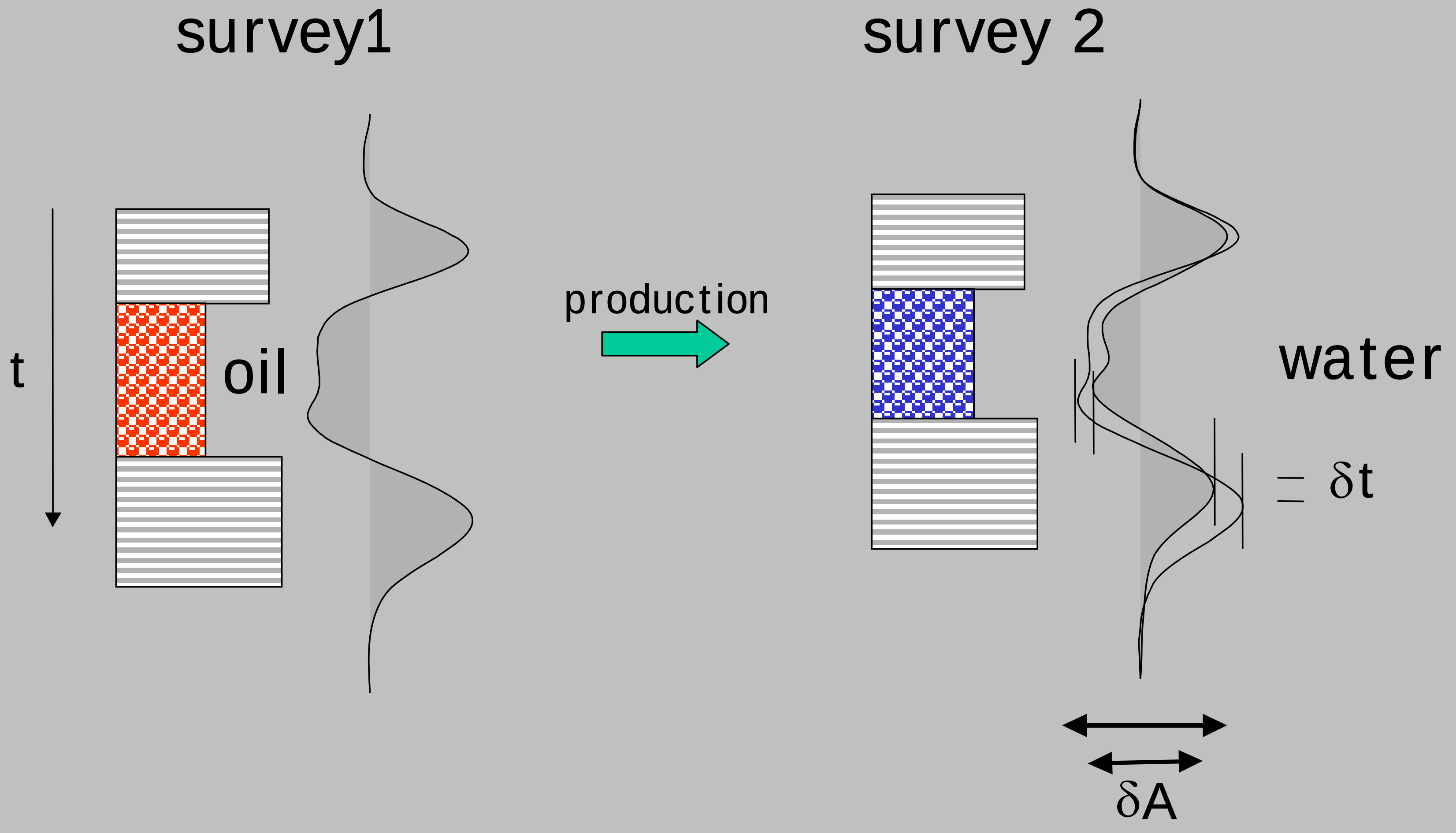
$$\left(\frac{1}{c^2}\partial_{tt}-\Delta\right)v=f \xrightarrow{\text{Fourierin time}} -\left(\frac{\omega^2}{c^2}+\Delta\right)\hat{v}=\hat{f}$$

- Discretization: $\boldsymbol{A\ v = F}$

final stage of ‘physics’:
expression in Numerical Linear Algebra

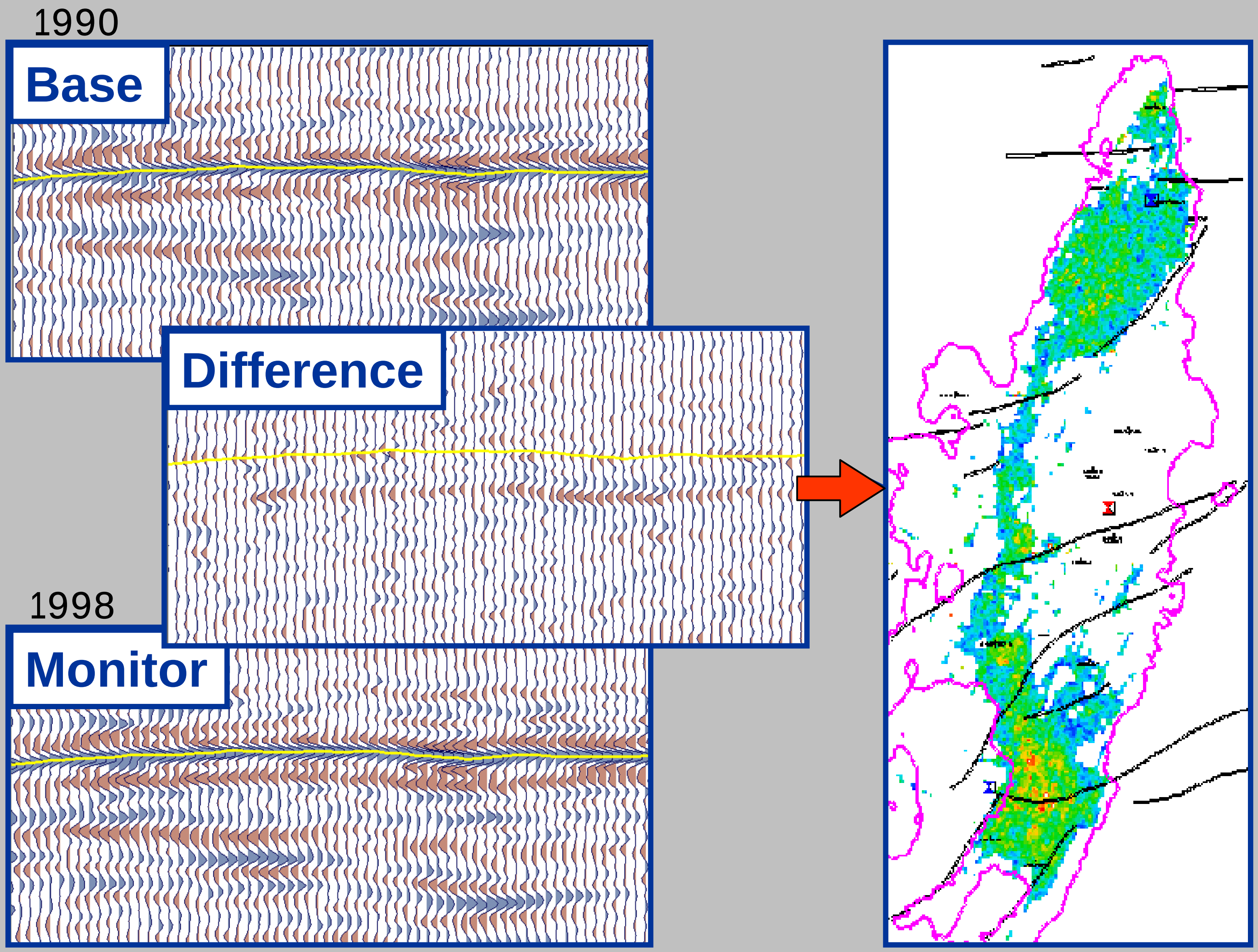
- Solution: $\boldsymbol{A = L\ U}$
- $\boldsymbol{L}$ is lower triangular, $\boldsymbol{U}$ is upper triangular
- back substitution: $\boldsymbol{v = U^{-1}L^{-1}F}$

The Geophysics of seismic 4D

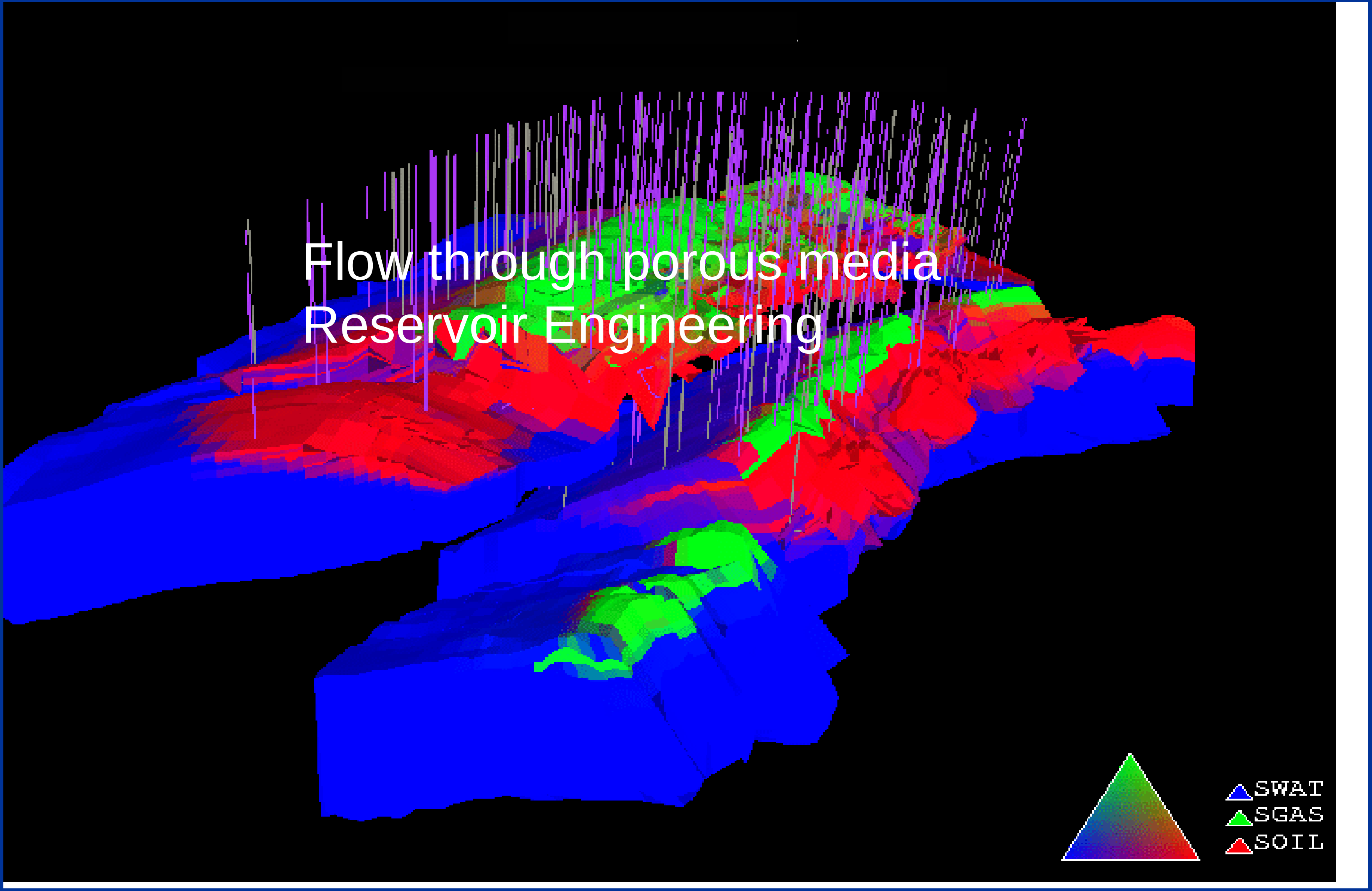


4D seismic amplitude and timing changes with production

Time-lapse Seismic

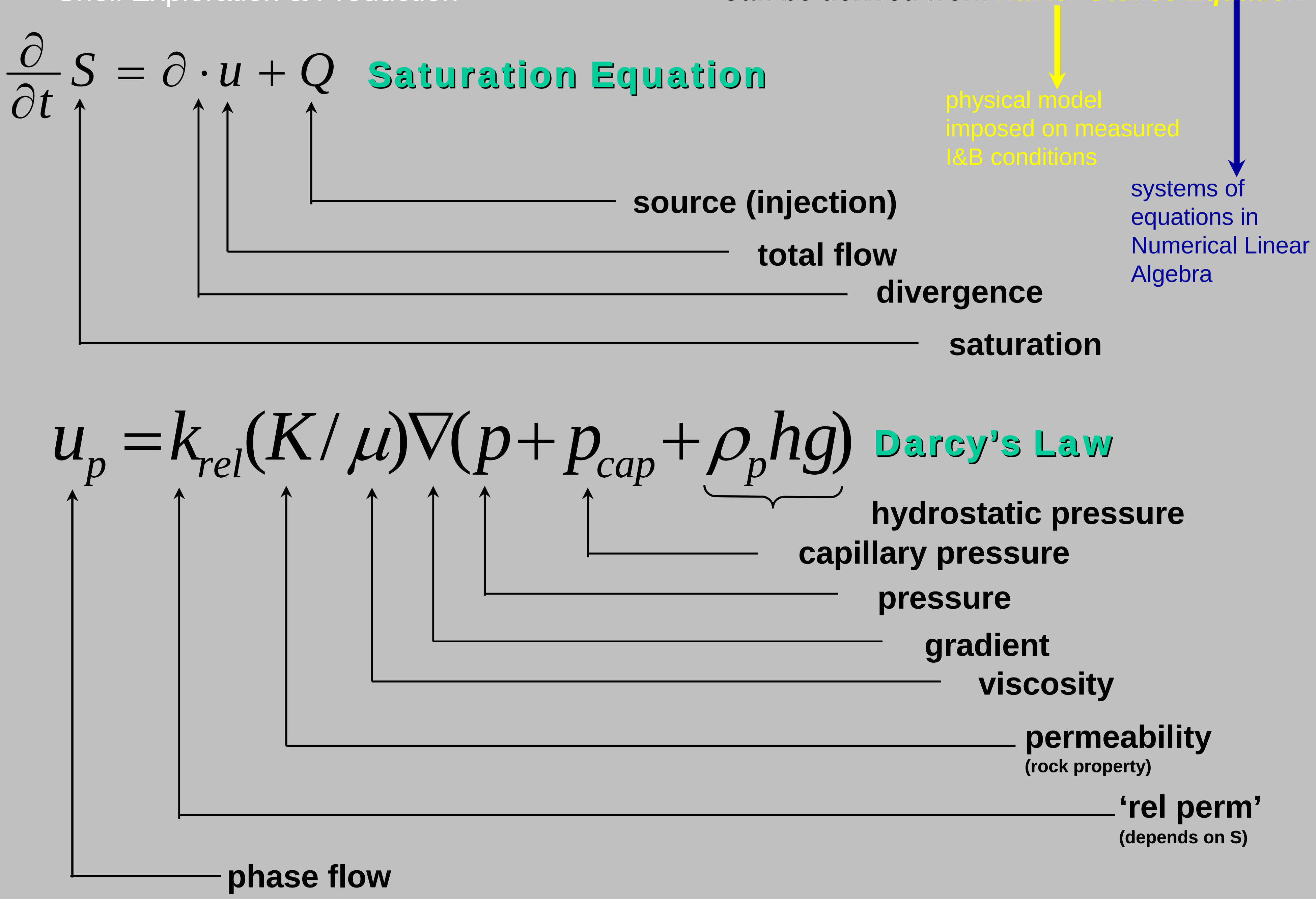


The Brent oil field in the North See



Flow through porous media
Reservoir Engineering

Flow Equations, to be solved numerically
Can be derived from *Navier Stokes Equation*



physical model
imposed on measured
I&B conditions

systems of
equations in
Numerical Linear
Algebra

$$\frac{\partial}{\partial t} S = \partial \cdot u + Q$$

Saturation Equation

source (injection)

total flow

divergence

saturation

$$u_p = k_{rel}(K/\mu)\nabla(p + p_{cap} + \underbrace{\rho_p hg})$$

Darcy's Law

hydrostatic pressure

capillary pressure

pressure

gradient

viscosity

permeability
(rock property)

'rel perm'
(depends on S)

phase flow

[illegible]

little success with ‘physical’ modeling

physical laws aim to describe the ‘internal state’

- relation original physical parameters and parameters in ‘numerical’ expressions difficult if not unknown
- physical objects – measurements, laws – are scale dependent
  - mismatch between scale_$_{\text{laws}}$ and scale_$_{\text{measurements}}$
- correctness of state description cannot be decided

Alternative:
construct physical models from observations

(planet orbits of Kepler and Gauss)

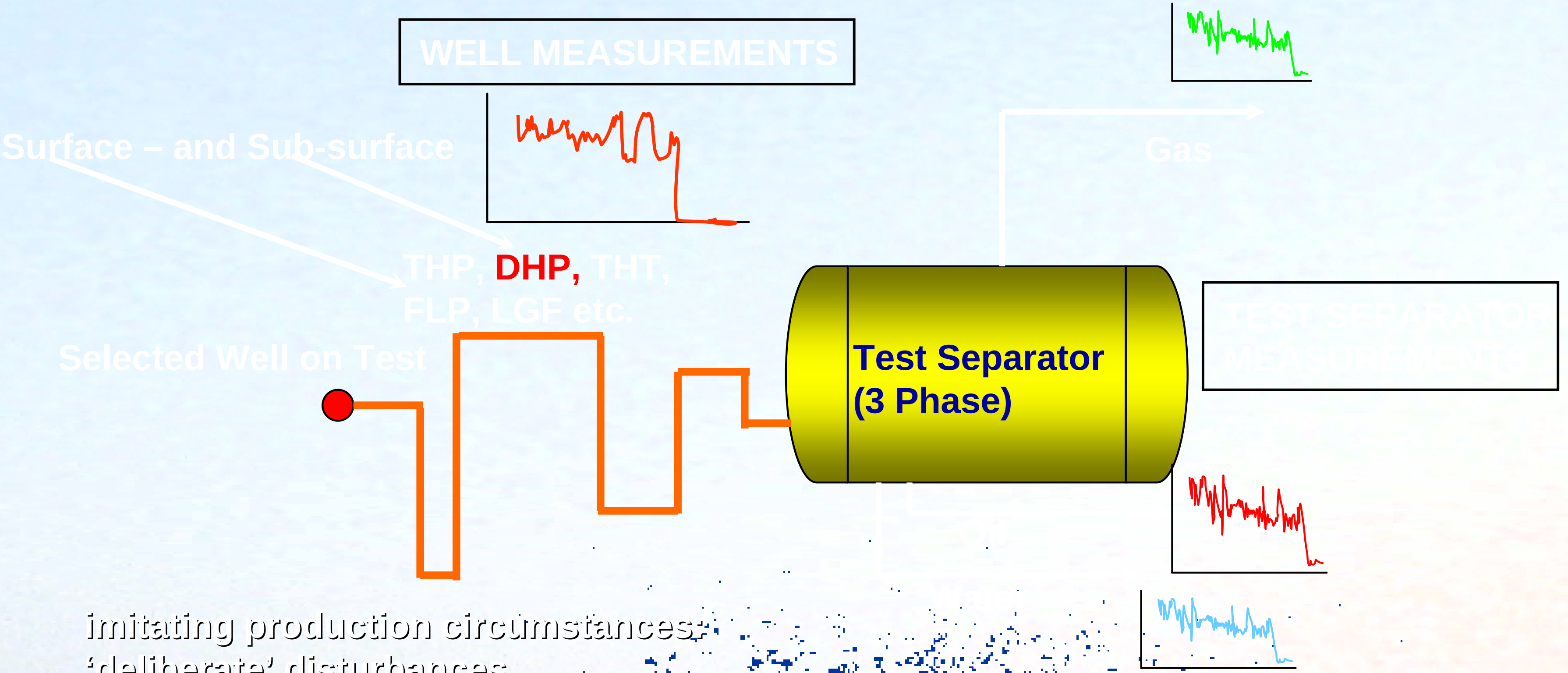
The ‘Champions Field’, South-Chinese Sea, offshore Brunei

transportation tubing

well heads

headers

separators



imitating production circumstances:
‘deliberate’ disturbances

Build Models using Well Test Data

$$\psi \in \mathbb{X}$$

$$\psi = \begin{pmatrix} q \\ u \end{pmatrix} \begin{matrix} \longleftarrow \textbf{well production} \\ \longleftarrow \textbf{input(s)} \end{matrix}$$

Assume that $\mathbb{X}$ is a compact metric space.

$\boxed{\text{Identify from well test experiment}}$ $\mathbf{F} : \mathbb{X} \to \mathbb{X}$

$$\mathbf{F}(\psi) = \begin{pmatrix} \mathbf{f}(\psi) \\ \underset{\uparrow \, \textbf{shift map}}{\mathbf{S}(u)} \end{pmatrix}$$

$\mathbf{F}$ viewed as dynamical system: ξ follows ψ if $\xi = \mathbf{F}^n(\psi)$ for some $n = 1, 2, \ldots$

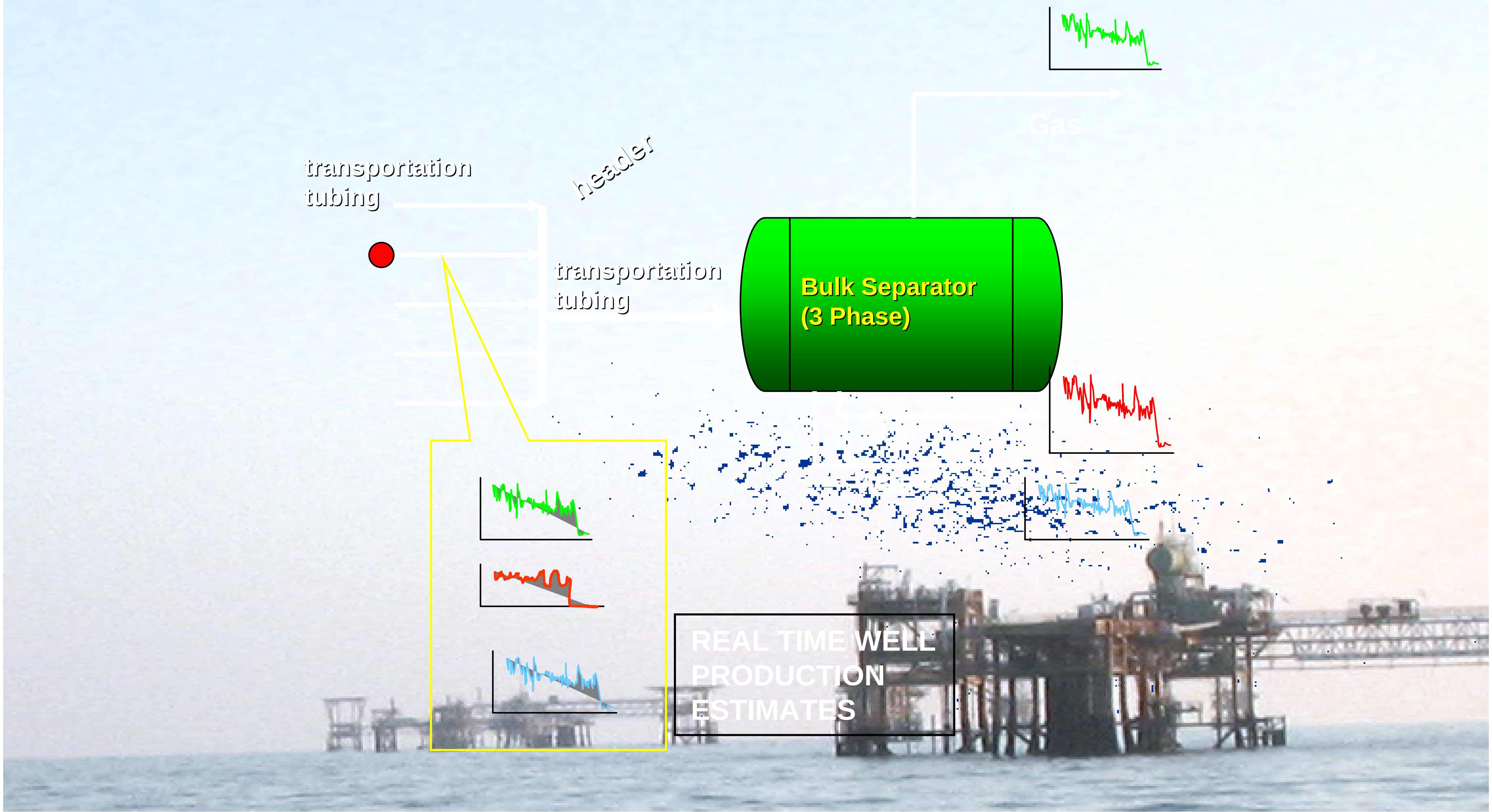
Identify $\mathbf{F}$ and its iterates with their graphs in $\mathbb{X} \times \mathbb{X}$.

$$\mathcal{O}\mathbf{F} = \bigcup_{n=1}^{\infty} \mathbf{F}^n$$

$$(\xi, \psi) \in \mathcal{O}\mathbf{F} \Leftrightarrow \xi = \mathbf{F}^n(\psi) \text{ for some } n = 1, 2, \ldots$$

$$\mathcal{O}\mathbf{F}(\psi) = \{\mathbf{F}(\psi), \mathbf{F}^2(\psi), \ldots\}$$ **positive orbit**

the orbits for each well give the production estimates for each well



transportation tubing

header

transportation tubing

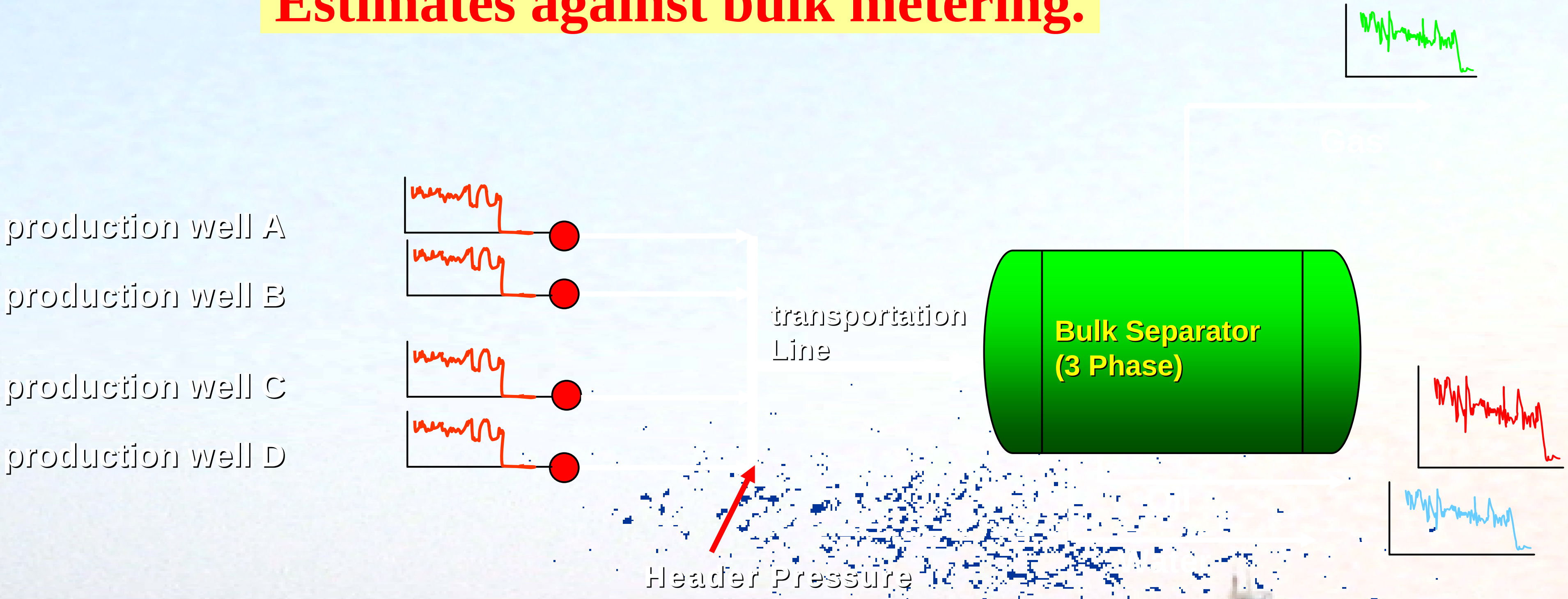
Bulk Separator (3 Phase)

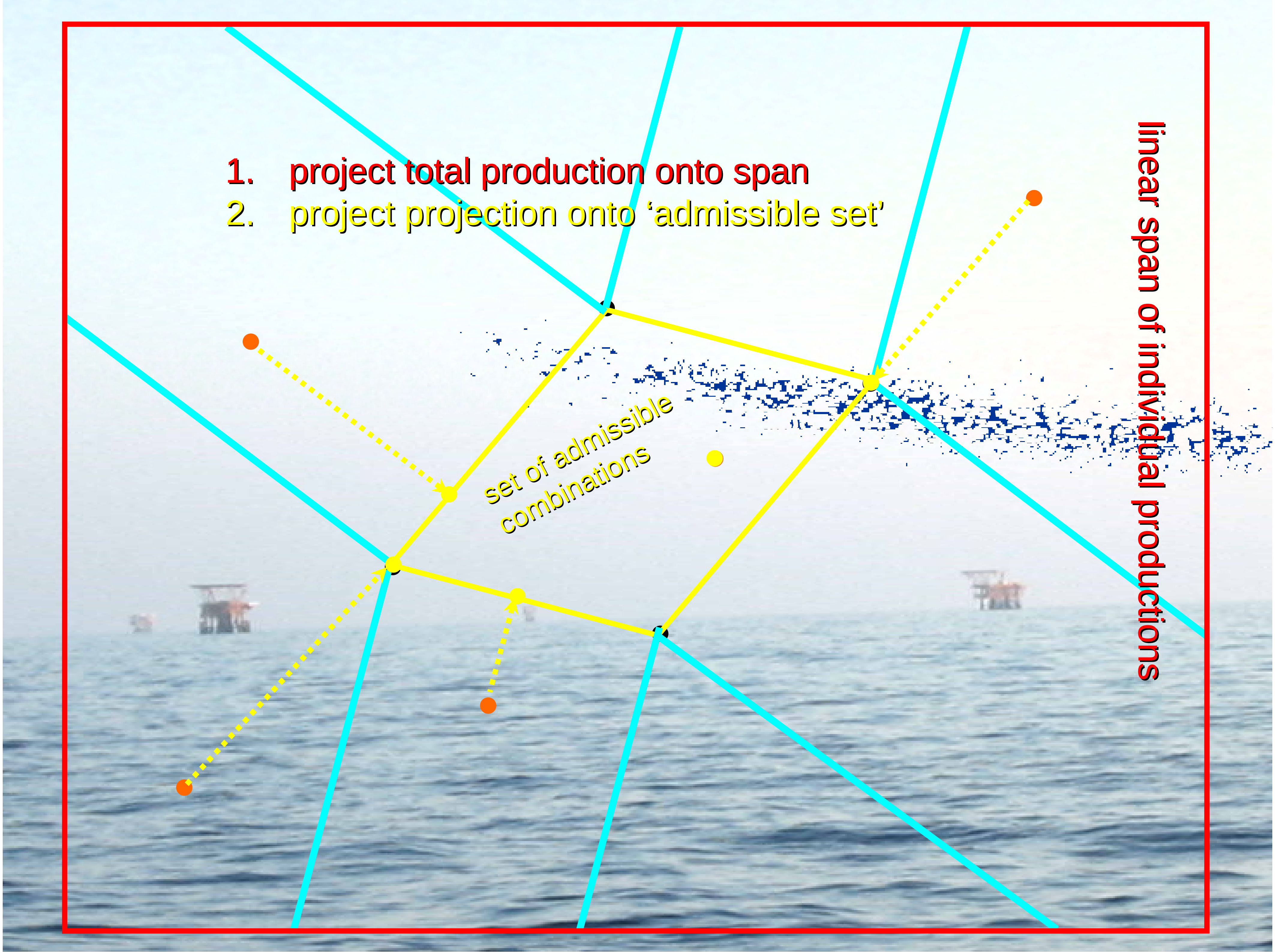
Gas

REAL TIME WELL PRODUCTION ESTIMATES

Daily Reconciliation

compare and adjust individual Estimates against bulk metering.





1. project total production onto span
2. project projection onto ‘admissible set’

dealing with uncertainty

$$\mathbb{V}_\epsilon = \{(\psi_1, \psi_2) \in \mathbb{X} \times \mathbb{X} \mid d(\psi_1, \psi_2) \leq \epsilon\}$$

signal-to-noise ratio

$$\epsilon\text{-chain } \{\psi_n\} : \psi_{n+1} \in \mathbb{V}_\epsilon(\mathbf{F}(\psi_n))$$

$$\mathcal{C}\mathbf{F} = \bigcap \{\mathcal{O}(\mathbb{V}_\epsilon \circ \mathbf{F}) \mid \epsilon > 0\}$$

(chain recurrent set)

$$\xi \in \mathcal{C}\mathbf{F}(\psi) \Leftrightarrow \forall \epsilon \; \exists \; \epsilon\text{-chain beginning at } \psi \text{ and ending at } \xi$$

best approximation

number of wells

bulk production

$$q \approx \sum_{j=1}^{N} \rho_j q_j$$

admissible linear combination

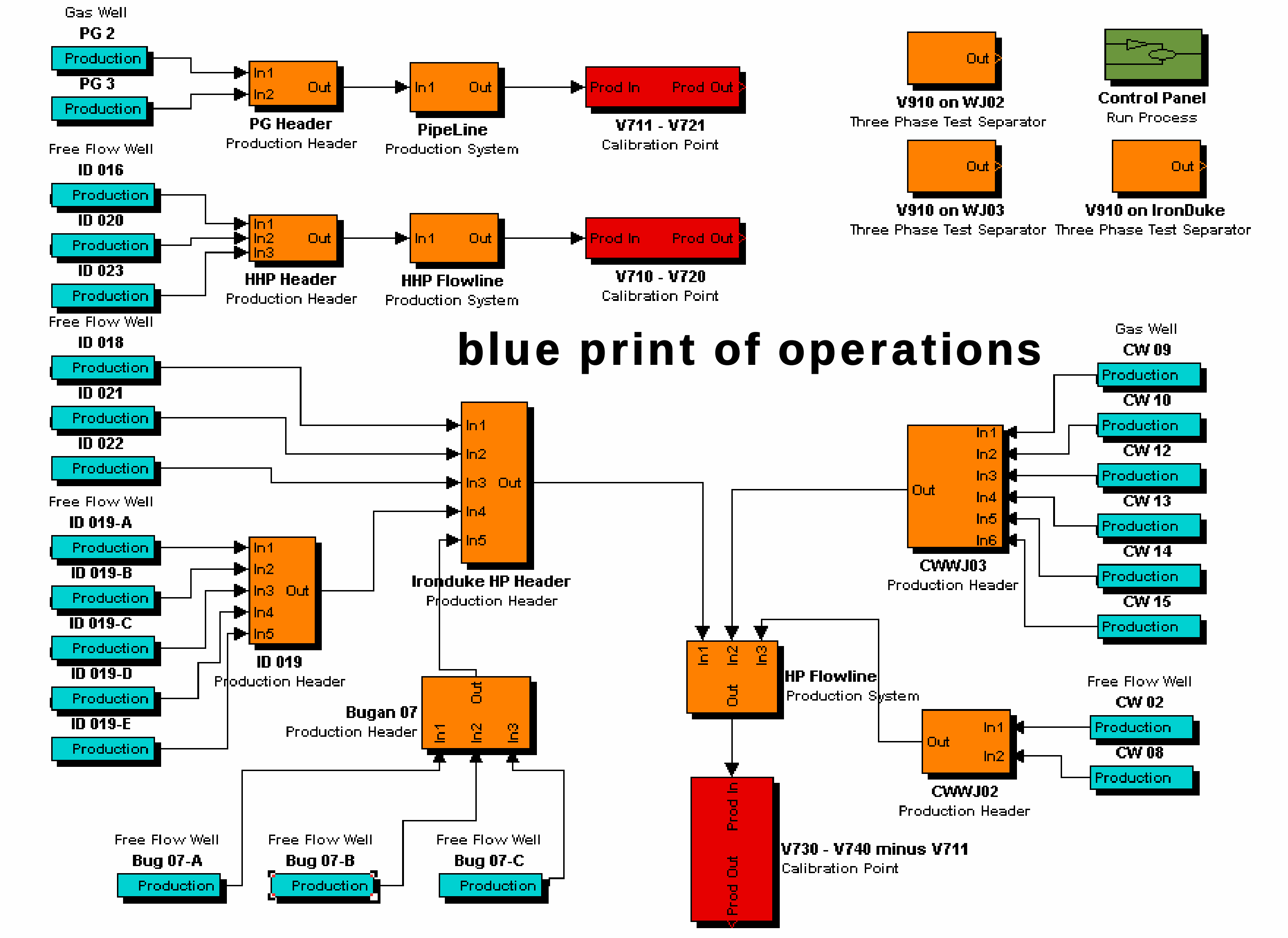
reconciliation coefficients

adapt

$$\hat{\psi}_j = \begin{pmatrix} \rho_j q_j \\ u \end{pmatrix}$$

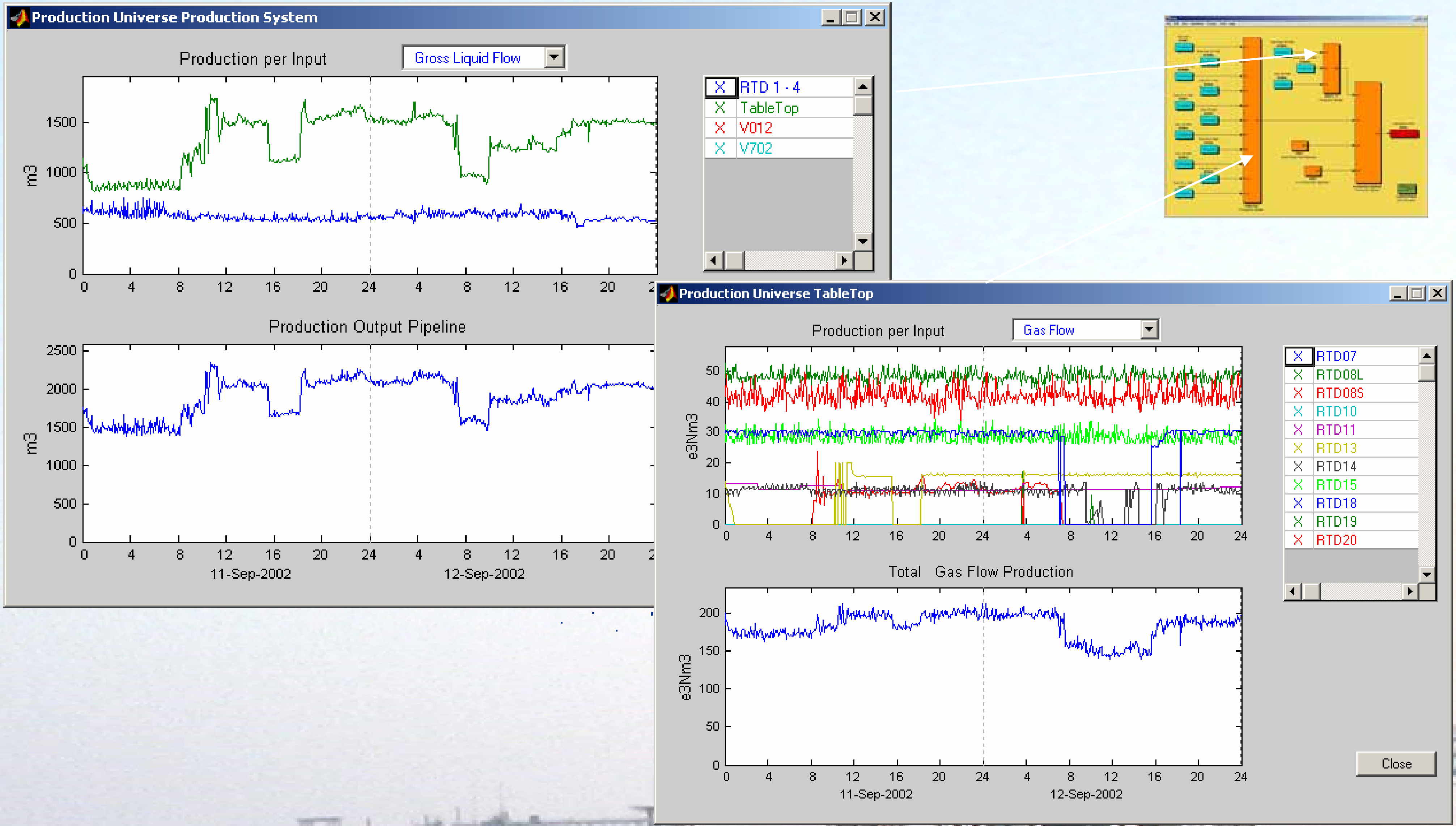
consistency $\mathcal{O}\mathbf{F}_j(\hat{\psi}_j) \subseteq \mathcal{C}\mathbf{F}_j(\psi_j)$ **continue** $\psi_j \longrightarrow \hat{\psi}_j$ **otherwise re-test**

Brunei – Iron Duke and Champion Oil Platforms



blue print of operations

blue print

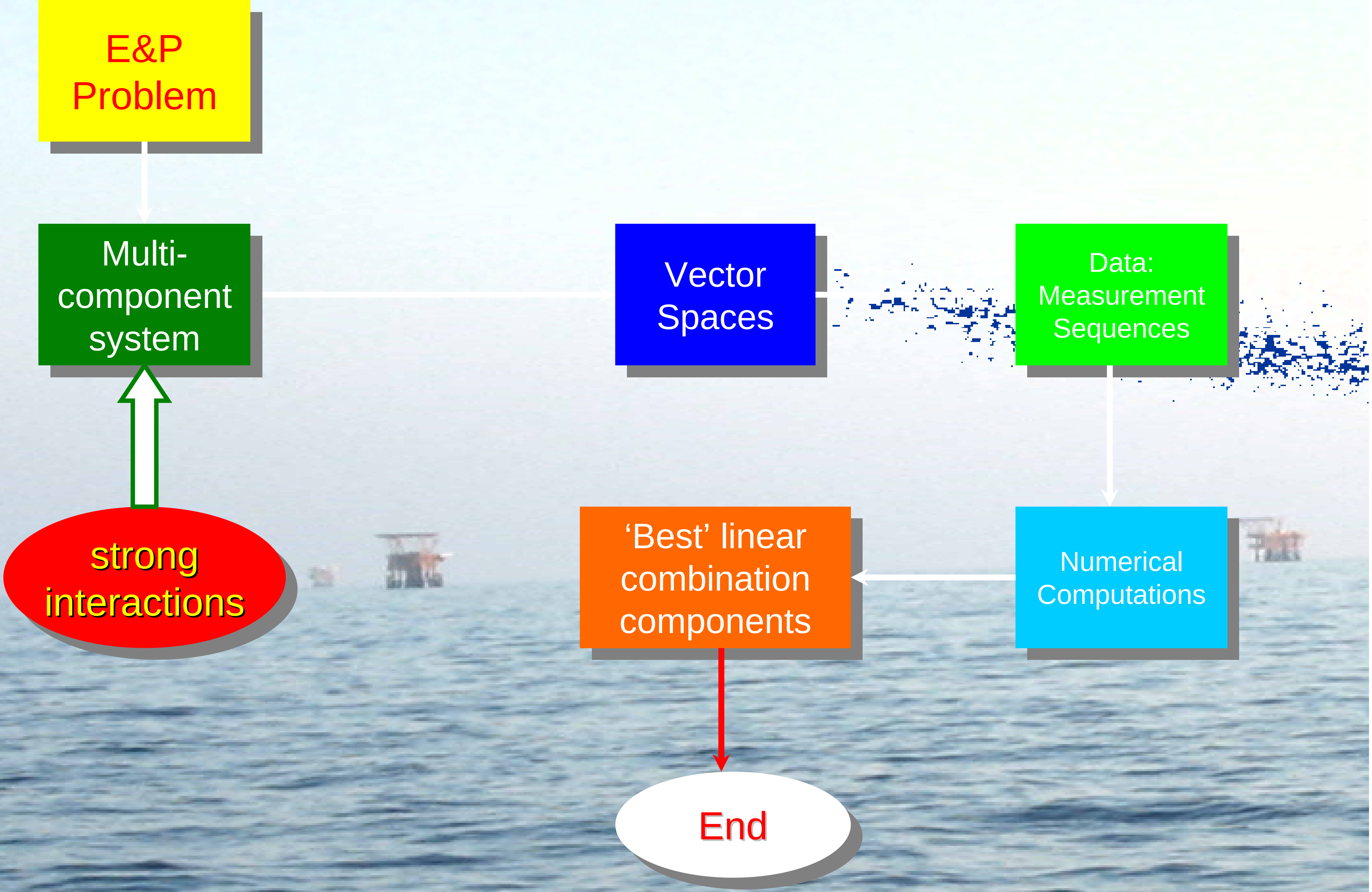


Production Universe Production System

Production per Input: Gross Liquid Flow

Production Universe TableTop

Production per Input: Gas Flow



E&P Problem

Multi-component system

strong interactions

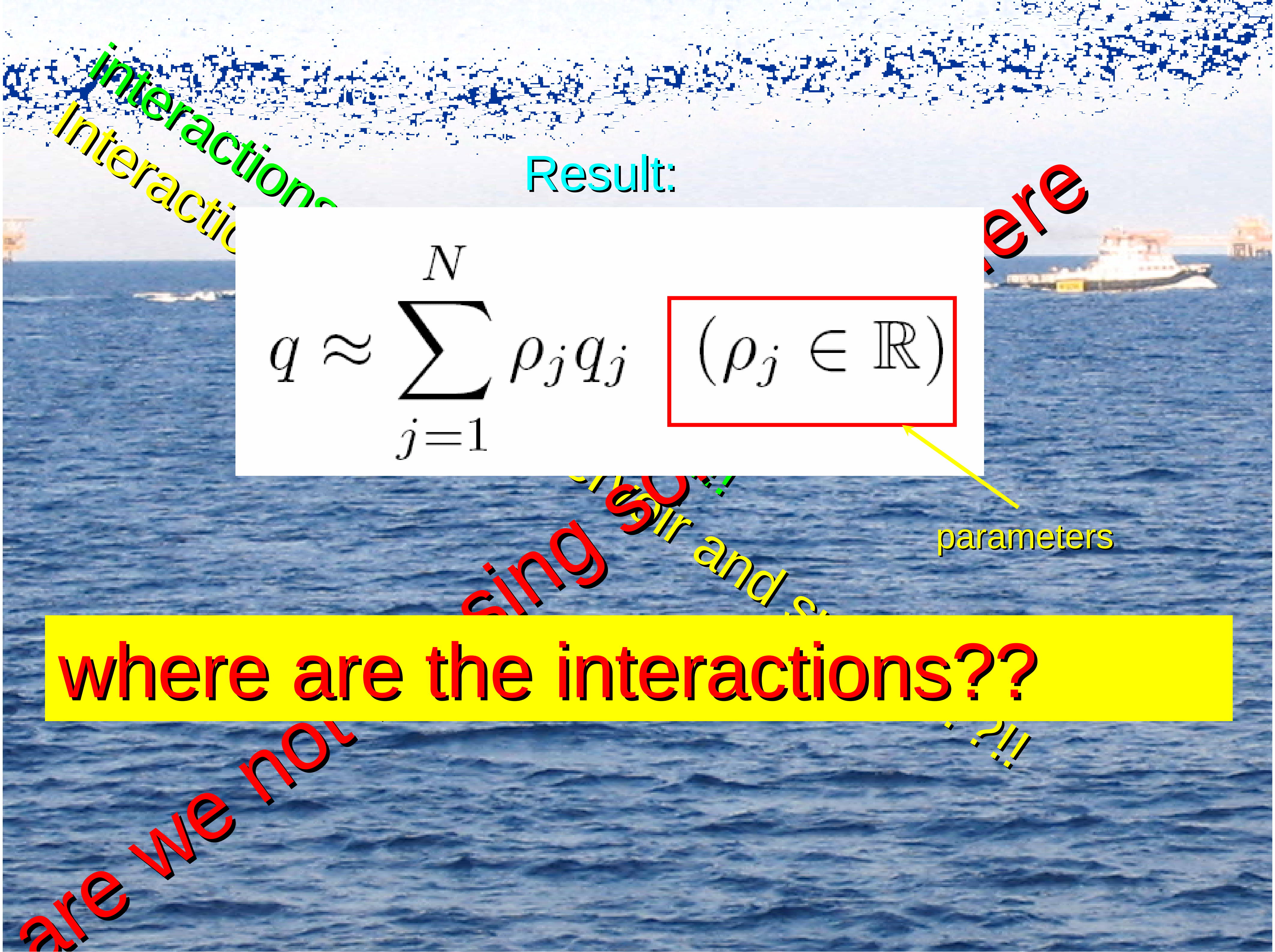
Vector Spaces

Data: Measurement Sequences

Numerical Computations

‘Best’ linear combination components

End



interactions

Interactio

ere

Result:

$$q \approx \sum_{j=1}^{N} \rho_j q_j \quad \boxed{(\rho_j \in \mathbb{R})}$$

parameters

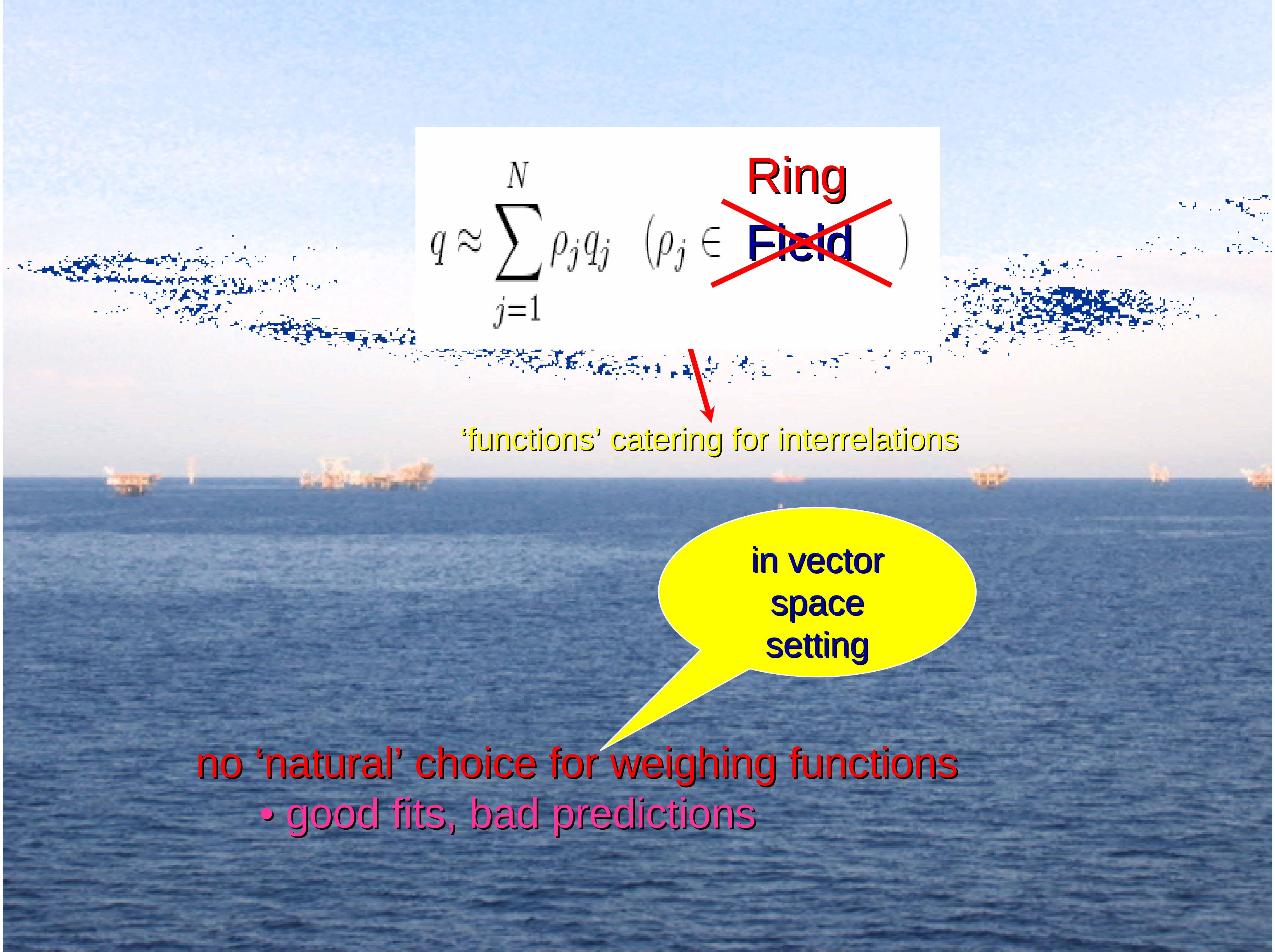
sing SO!!

eir and st

?!!

are we not

where are the interactions??



$$q \approx \sum_{j=1}^{N} \rho_j q_j \quad \left(\rho_j \in \overset{\text{Ring}}{\cancel{\text{Field}}}\right)$$

'functions' catering for interrelations

in vector space setting

no 'natural' choice for weighing functions

- good fits, bad predictions

$$q \approx \sum_{j=1}^{N} \rho_j q_j \quad \left(\rho_j \in \overset{\text{Ring}}{\xcancel{\text{Field}}}\ \right)$$

Polynomial Ring

casting the ‘*Decomposition Problem’* in terms of Polynomial Rings:

$$\mathbf{R} = \mathbb{R}[x_1, \ldots, x_n]$$

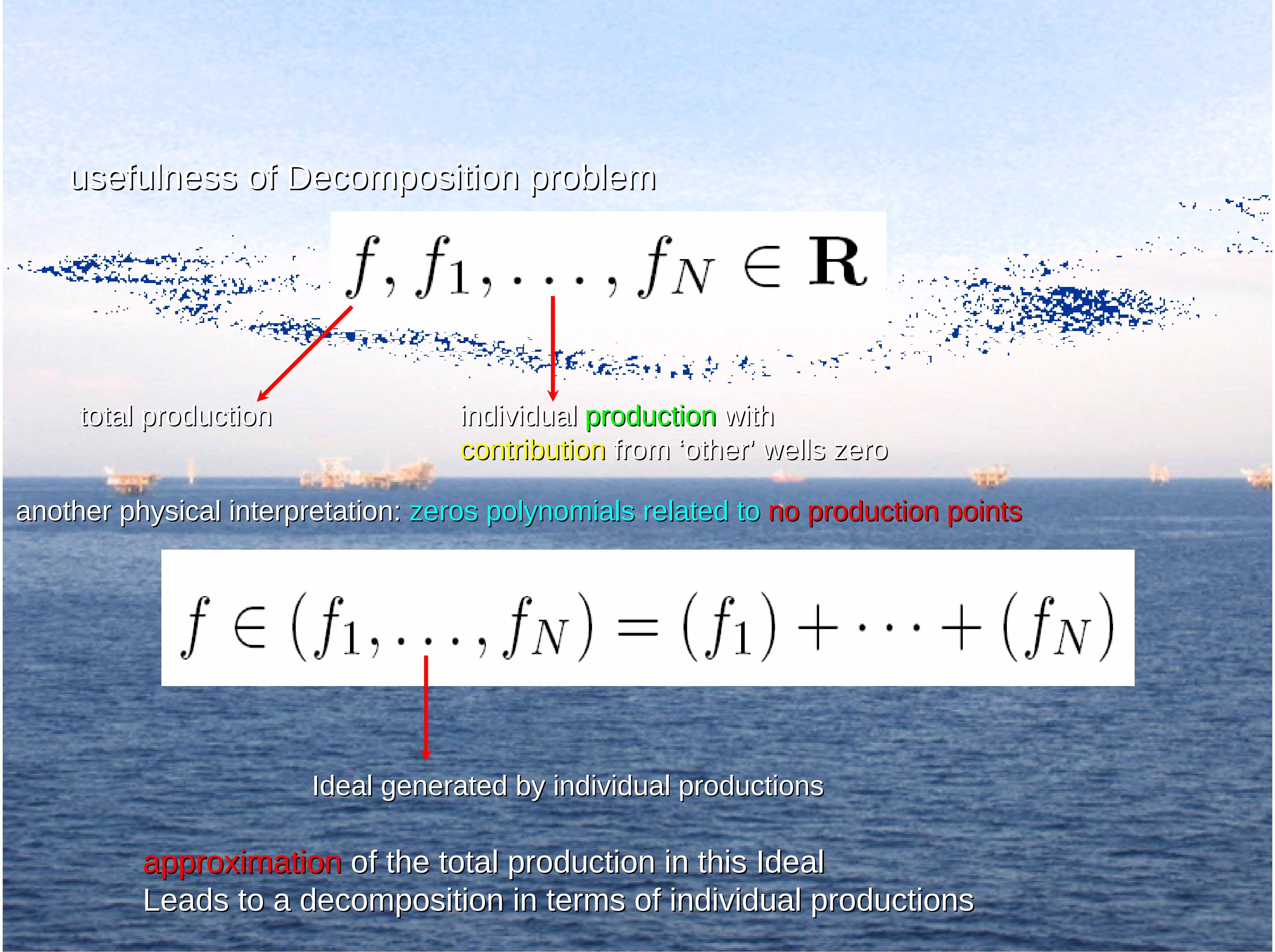
“physical meaning associated with indeterminates”:

$$\begin{array}{lcl} \psi_t & : & \mathbb{R}[x_1, \ldots, x_n] \to \mathbb{R} \\ t & \in & \{1, \ldots, T\} \subset \mathbb{N} \quad (\textit{sample number}) \end{array}$$

substitution homomorphism

then e.g. $\psi_t(x_1) = THP_t$ ⇒ ordering choice has physical consequences

$$\mathbb{R}[THP, FLP, \ldots]$$



usefulness of Decomposition problem

$$f, f_1, \ldots, f_N \in \mathbf{R}$$

total production

individual production with
contribution from ‘other’ wells zero

another physical interpretation: zeros polynomials related to no production points

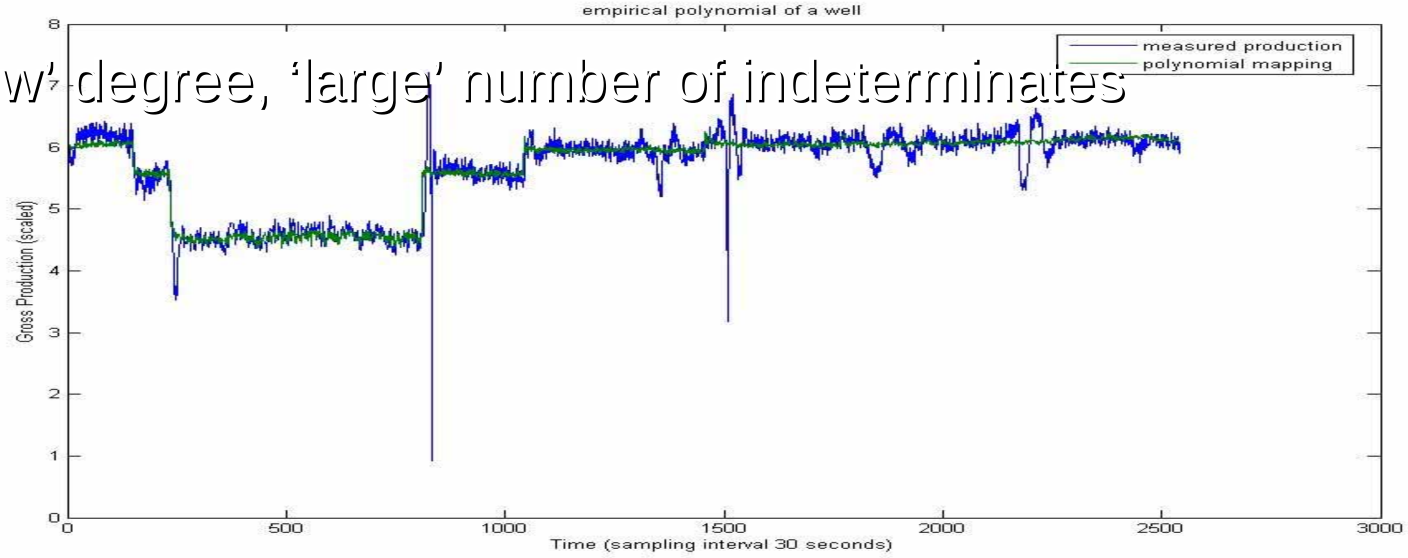
$$f \in (f_1, \ldots, f_N) = (f_1) + \cdots + (f_N)$$

Ideal generated by individual productions

approximation of the total production in this Ideal
Leads to a decomposition in terms of individual productions

example ***empirical*** polynomial

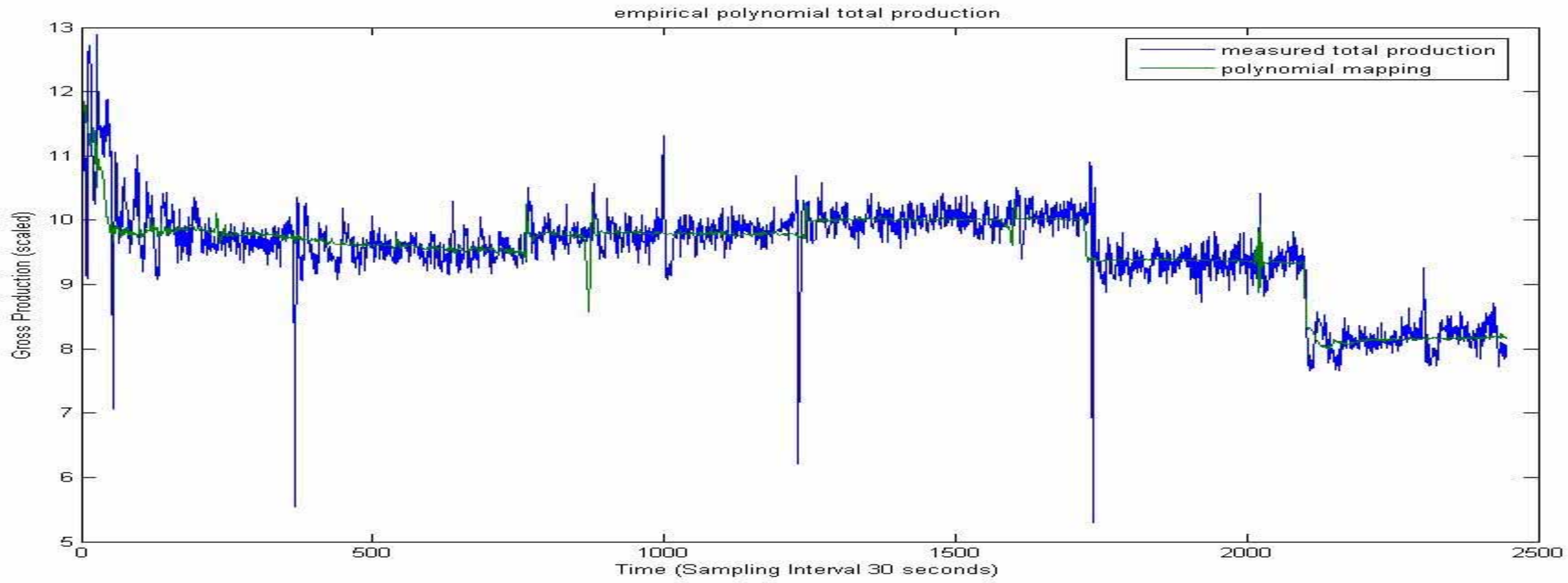
$$\begin{gathered}
f_1 \approx 2.9232 * (\sqrt{((DHP_1 - DHP_{tu}) * DHP_{tu})} * FLP) - \\
2.7075 * (\boxed{\sqrt{((DHP_1 - DHP_{tu}) * DHP_{tu})}} * DHP_1) + \\
2.9517 * \sqrt{((DHP_{tu} - THP) * THP)} - \\
1.2158 * (\sqrt{((DHP_{tu} - THP) * THP)} * THP) + \\
0.3856 * (\sqrt{((THP - FLP) * FLP)} * DHP_{tu}) + \\
2.5594 * (\sqrt{((DHP_1 - DHP_{tu}) * DHP_{tu})} * DHP_{tu})
\end{gathered}$$



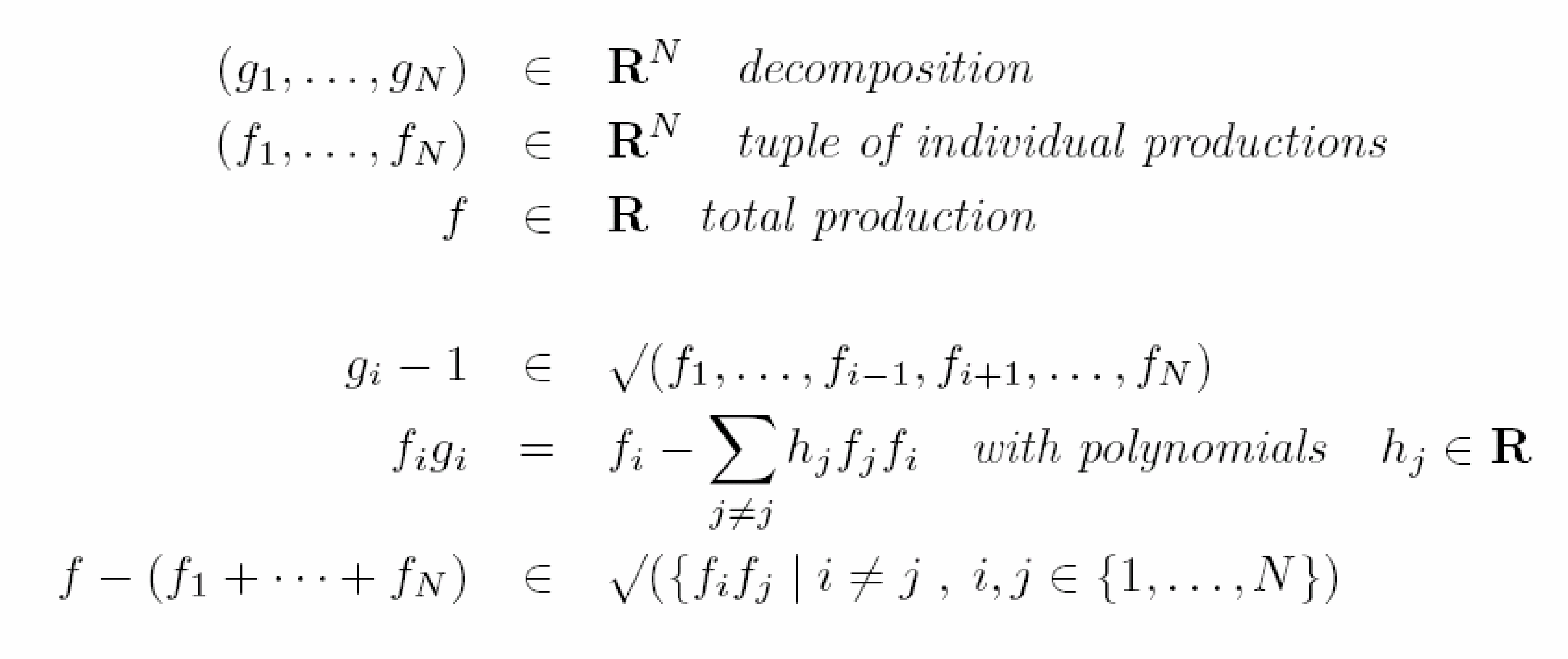
‘low’ degree, ‘large’ number of indeterminates

example: empirical polynomial total production

$$\begin{gathered}
f \approx 0.5113 * (f_1 * DHP_{tu} * DHP_2) - \\
1.2104 * (f_1 * FLP * \surd((DHP_{tu} - THP) * THP)) + \\
0.4759 * (f_2 * \surd((DHP_1 - DHP_{tu}) * DHP_{tu}) * \surd((DHP_1 - DHP_{tu}) * DHP_{tu})) - \\
0.4071 * (f_1 * DHP_{tu} * DHP_1) + \\
0.0841 * (f_1 * THP * THP) - \\
0.2738 * (f_2 * \surd((DHP_1 - DHP_{tu}) * DHP_{tu}) * \surd((DHP_1 - DHP_{tu}) * DHP_{tu}))
\end{gathered}$$



breaking open the interrelationships problem:
on-going research, under debate



$$
\begin{aligned}
(g_1, \ldots, g_N) &\in \mathbf{R}^N \quad \textit{decomposition} \\
(f_1, \ldots, f_N) &\in \mathbf{R}^N \quad \textit{tuple of individual productions} \\
f &\in \mathbf{R} \quad \textit{total production}
\end{aligned}
$$

$$
\begin{aligned}
g_i - 1 &\in \sqrt{}(f_1, \ldots, f_{i-1}, f_{i+1}, \ldots, f_N) \\
f_i g_i &= f_i - \sum_{j \neq j} h_j f_j f_i \quad \textit{with polynomials} \quad h_j \in \mathbf{R} \\
f - (f_1 + \cdots + f_N) &\in \sqrt{}(\{f_i f_j \mid i \neq j \ , \ i, j \in \{1, \ldots, N\}\})
\end{aligned}
$$

starting activities of an ambitious Research Program

In 2004 e-mail contact with Prof. Martin Kreuzer

Algebraic Subject	E & P Application
[illegible]	**Interrelationships** Sub-surface – Surface Relationships, Ultimate Recovery
[illegible]	[illegible] [illegible] gravity. Good [illegible] and exploration [illegible]
Elimination Theory	**www2** Acronym for: Where, when, what to measure. Minimal requirements [illegible] infra structure.
Invariant Theory	**Generic elements** Global exchange of Information
Homotopy	**Test versus Production/Short-term versus Long-term** The changes are viewed as continuous deformations. Of importance for Starting-up sequences and Ultimate Recovery
Automated Theorem Proving	**Diagnostics and Decisions** Including relationships between processes that run on different time scales, e.g. early recognition of building-up water break through. Subject may be considered as next generation Artificial Intelligence.
Computational Homology	**Surface characterization** Surface characterization of sub-surface through computation of homology groups. Of particular importance for last pair.
D - Modules	**Non-seismic Exploration** This application is possible since this algebraic subject allows the consideration of spatial variation. This pair is coupled with the first – and second pair.

The CoCoA Research Agreement

university
of Dortmund

university
of Genova

sponsored by the Shell Research Foundation

Shell Exploration & Production

Explaining our program to the algebraic community at CoCoA Summer School

Algebraic Subject	E & P Application
[illegible]	**Interrelationships** Sub-surface <-> Surface Relationships: Ultimate Recovery
[illegible]	[illegible]
Elimination Theory	**www2.m** Acronym for 'Where, when, what to measure'. Minimal requirements technical infra structure.
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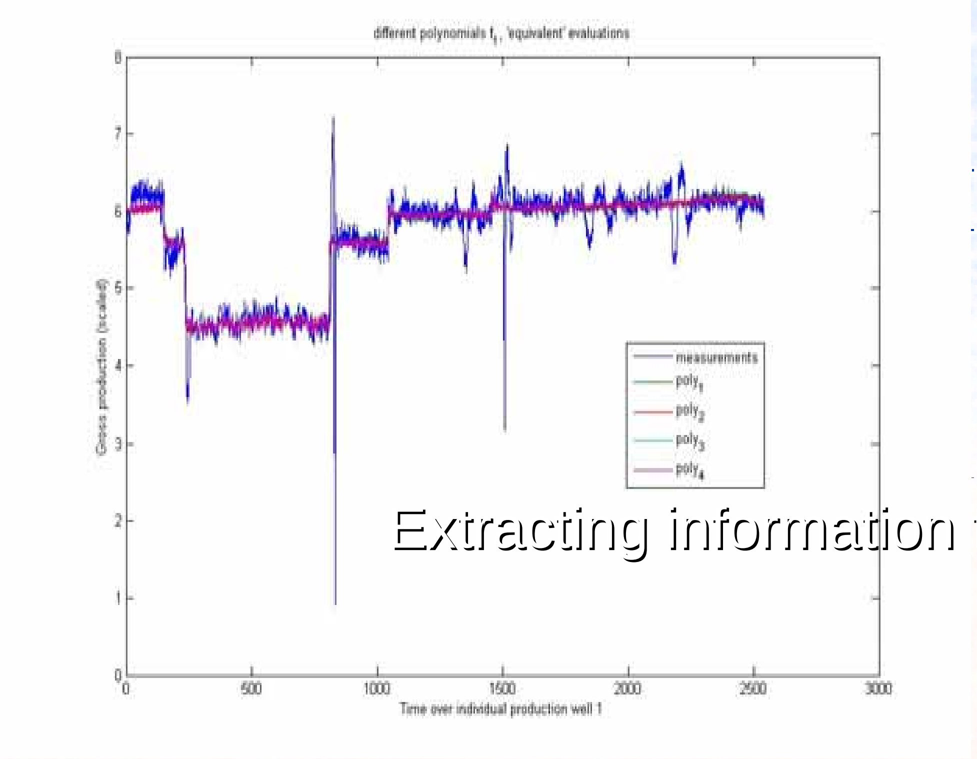
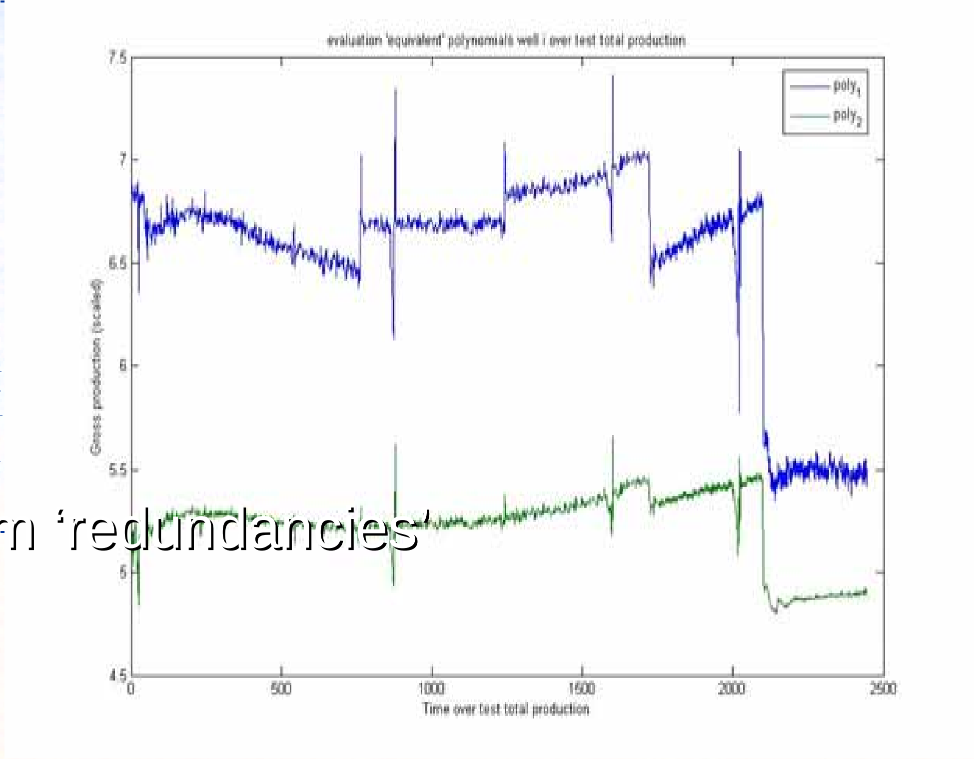
Extracting information from ‘redundancies’





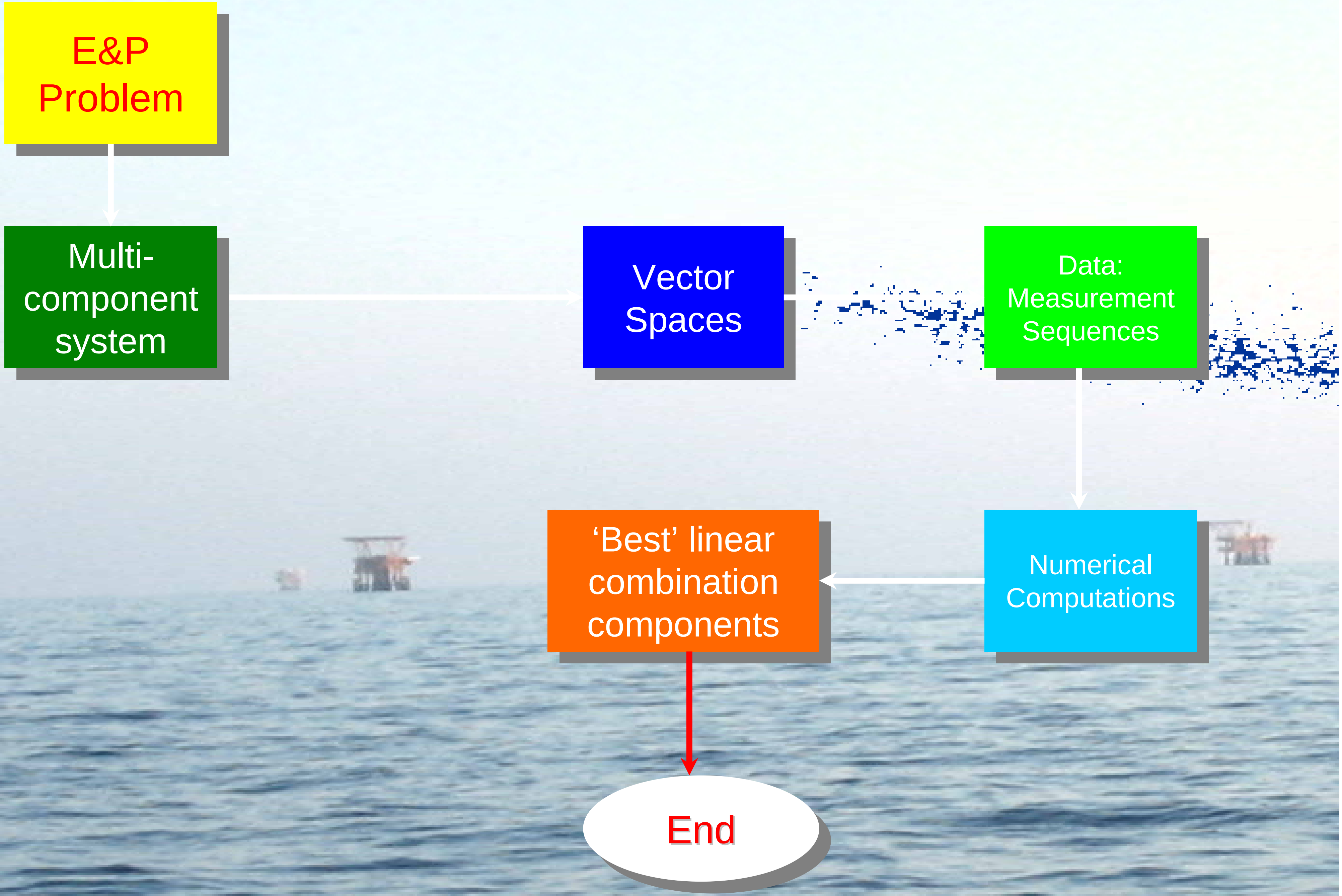
$$\psi_\lambda \quad : \quad \mathbb{R}[x_1, \ldots, x_n] \rightarrow \mathbb{R}$$

$$\lambda \quad \in \quad \textit{Sampling points operating range of individual production of well } i$$

$$f_{i_k} \quad , \quad f_{i_l} \quad \textit{alternative polynomial representations for individual production of well } i$$

$$f_{i_k} - f_{i_l} \quad \in \quad Ker(\psi_\lambda)$$

$$Ker(\psi_\lambda) \quad : \quad \textit{Ideal of relations satisfied by } \lambda \textit{ in } \mathbf{R}$$



E&P Problem

Multi-component system

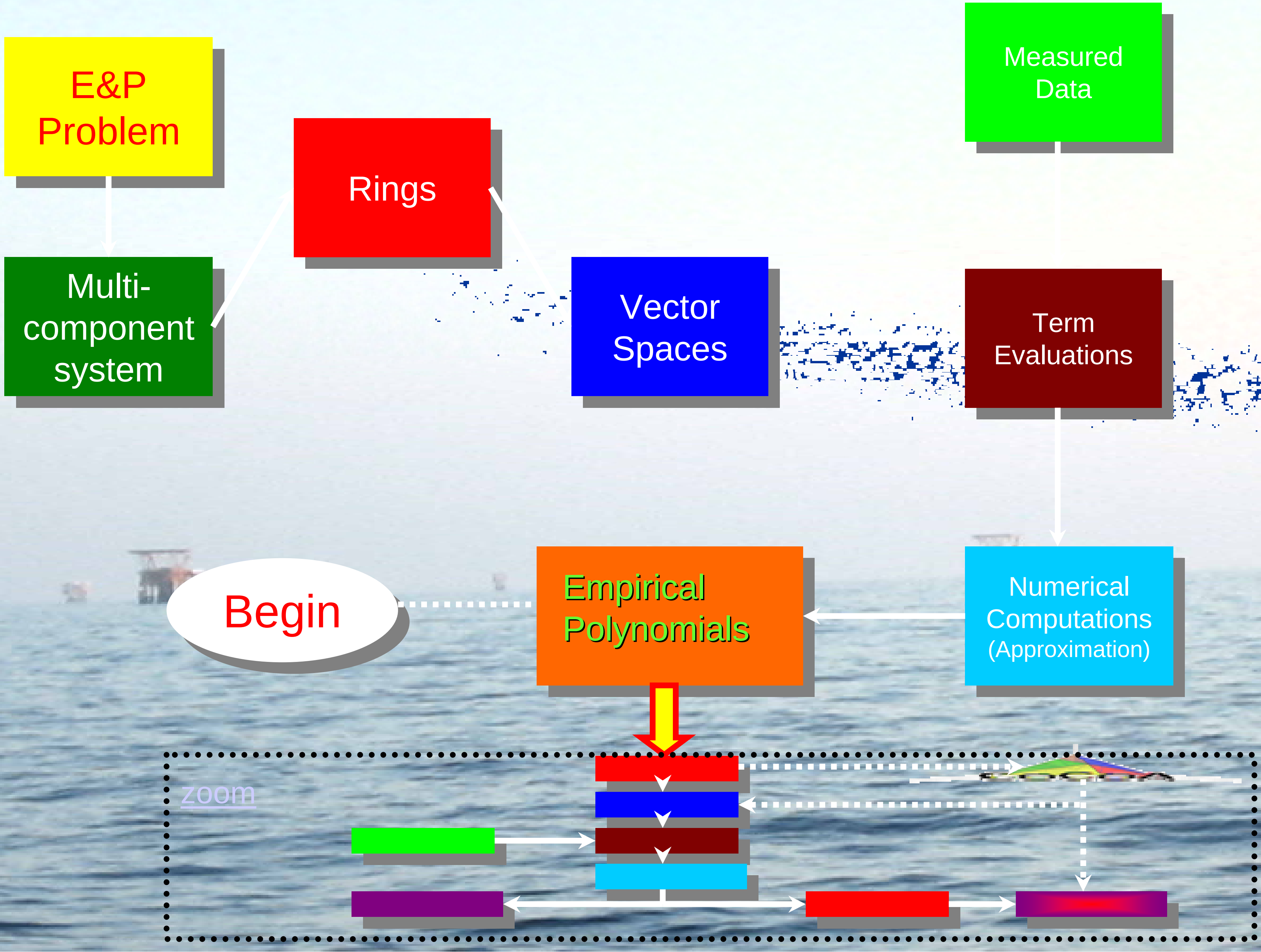
Vector Spaces

Data: Measurement Sequences

Numerical Computations

‘Best’ linear combination components

End



E&P Problem

Multi-component system

Rings

Vector Spaces

Measured Data

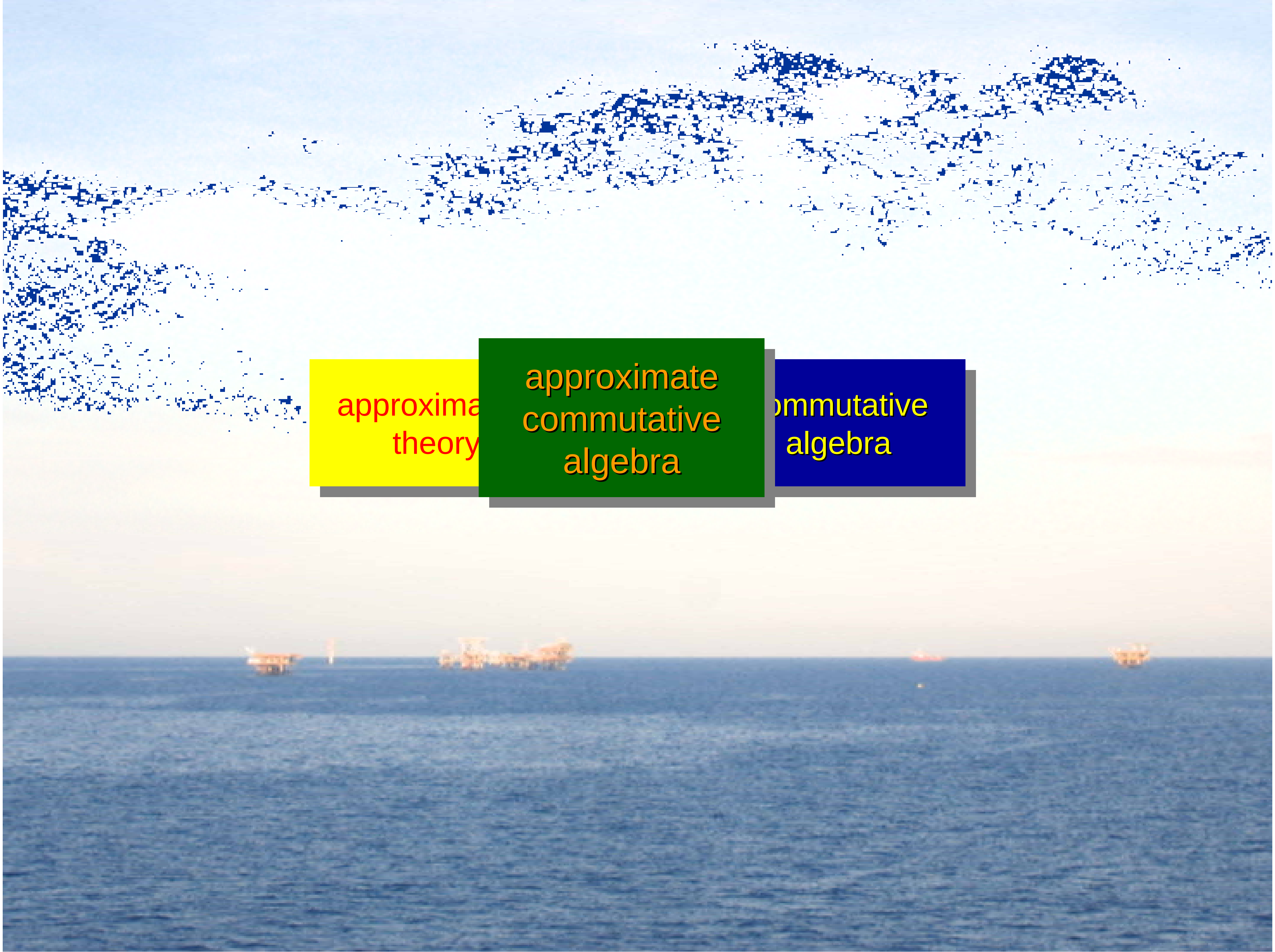
Term Evaluations

Numerical Computations (Approximation)

Empirical Polynomials

Begin

zoom



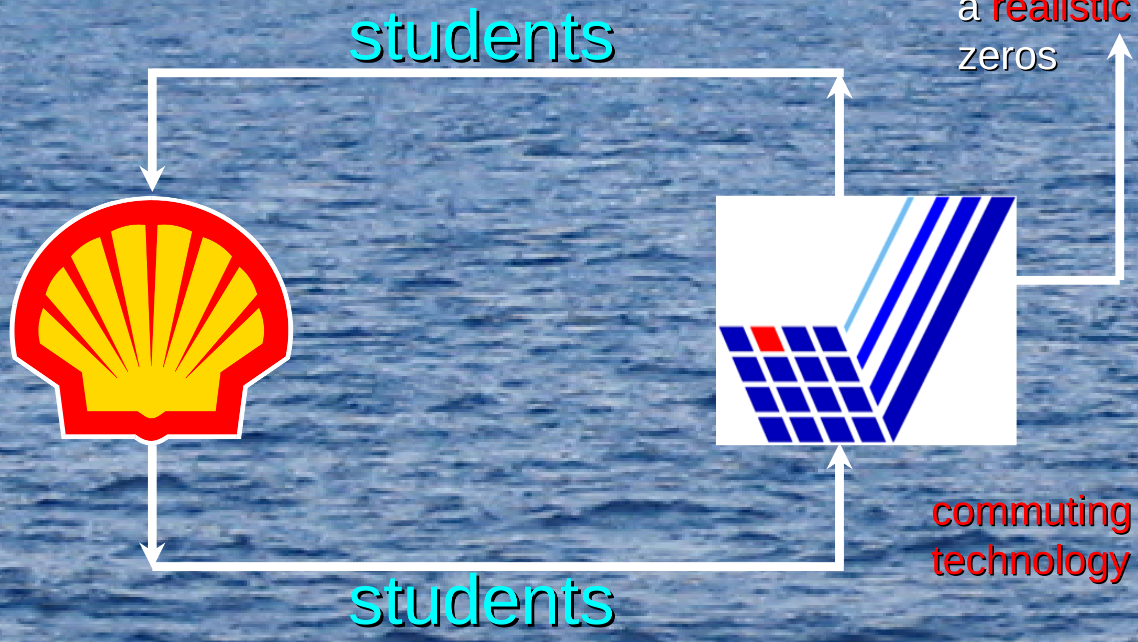
approxima
theory

approximate
commutative
algebra

ommutative
algebra

spreading knowledge within the cooperation:

within the international CoCoA team



BM algorithm:
to estimate an ideal for
a realistic set of 'perturbed'
zeros

students

direct confrontation
with technology

commuting between
technology and mathematics

students

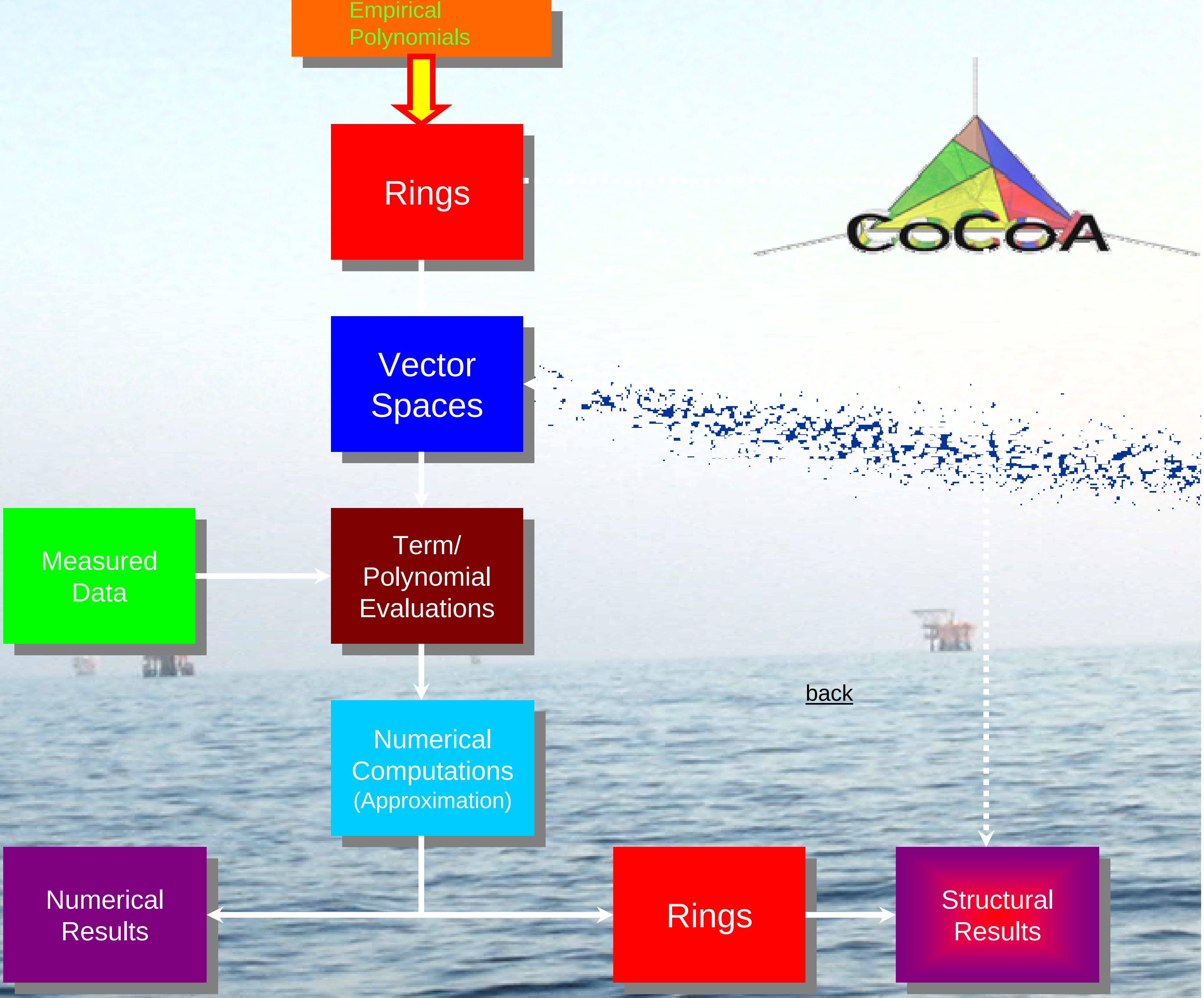
First student: Daniel Heldt
Next student: Matthias Machnik

Everyone going offshore has to pass the ‘HUET’ training

encountering
basic engineering

also that can have its beauties ….

Thank you! !



Empirical Polynomials

Rings

Vector Spaces

Measured Data

Term/ Polynomial Evaluations

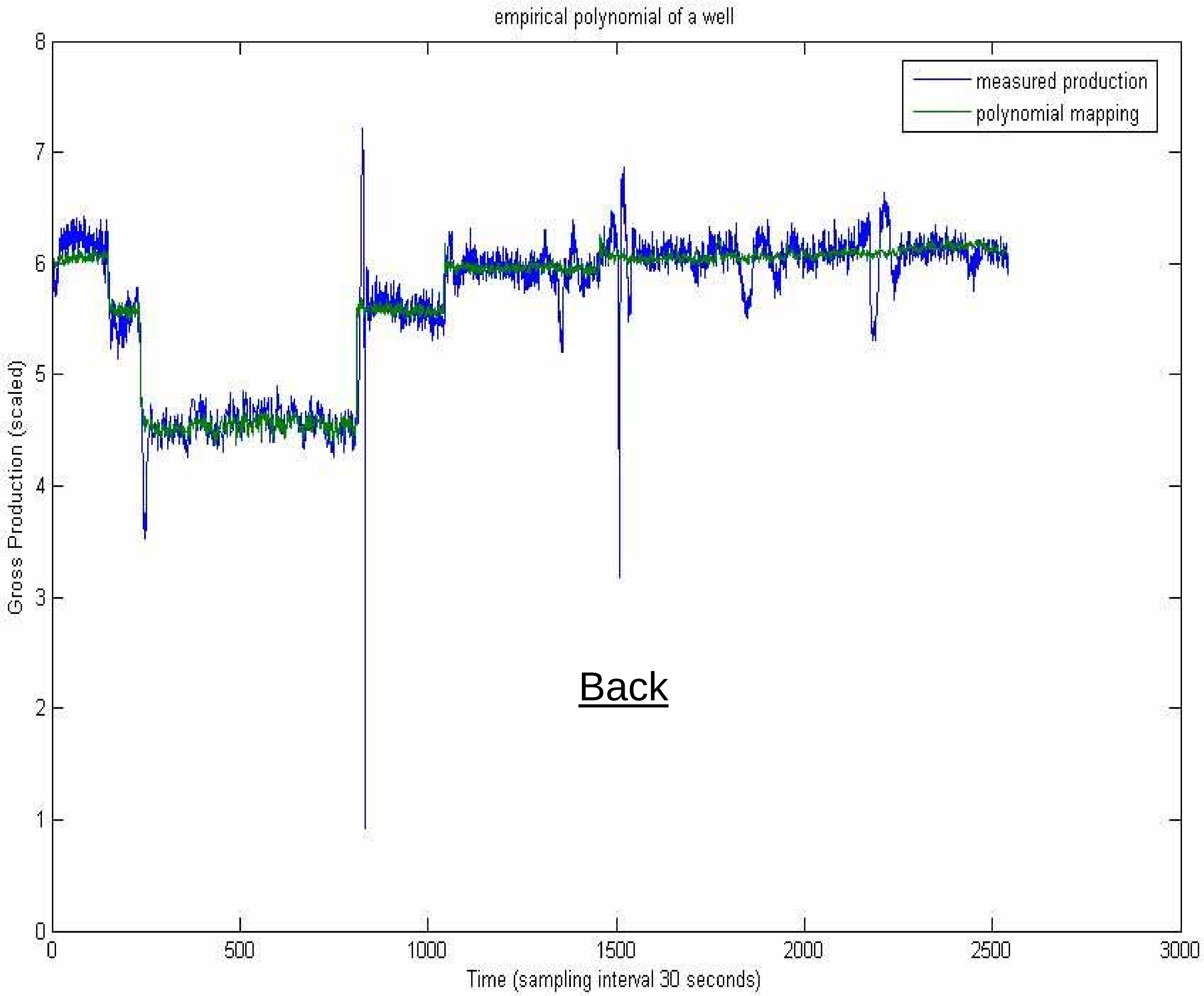
Numerical Computations (Approximation)

Numerical Results

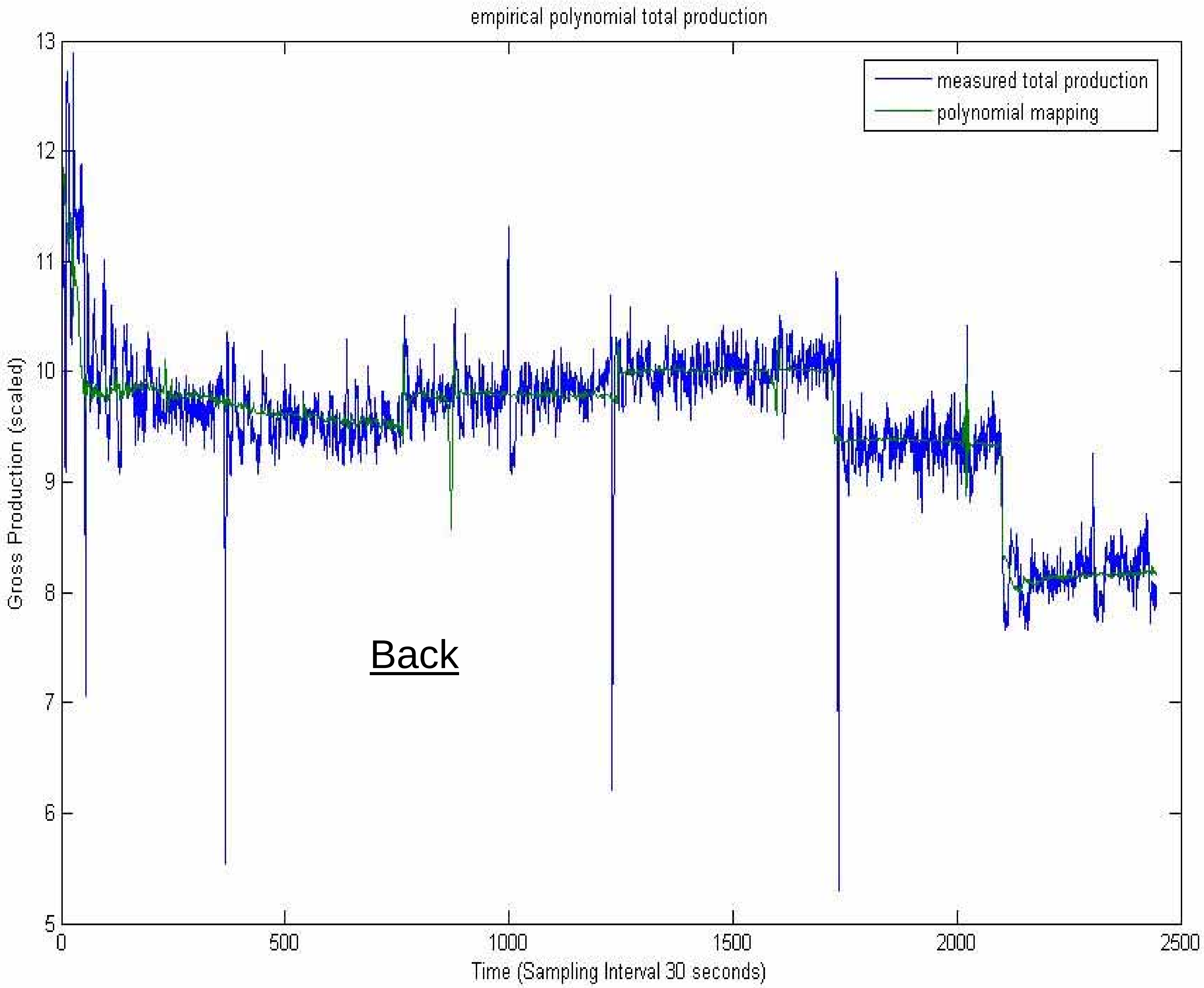
Rings

Structural Results

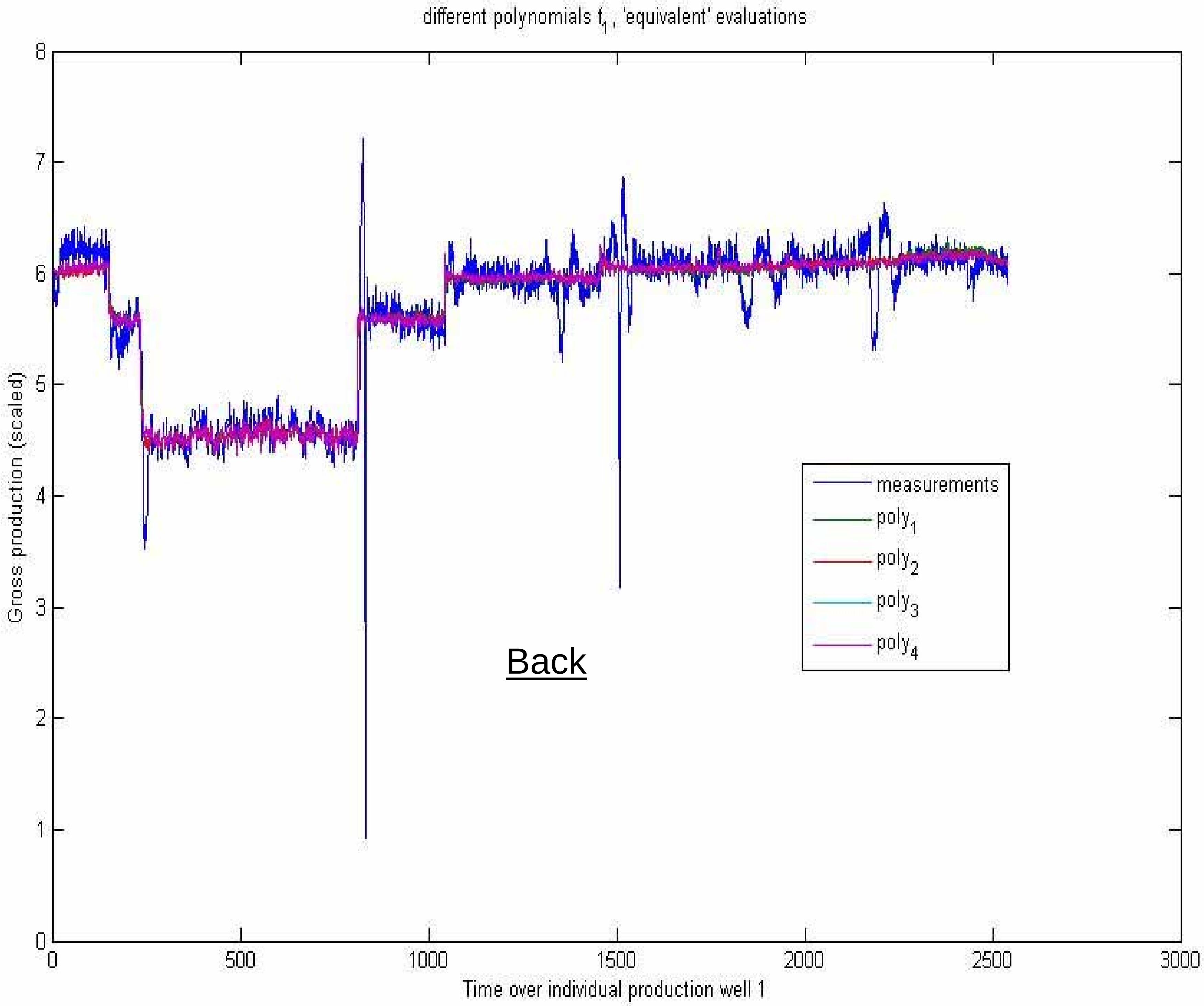
back


empirical polynomial of a well
measured production
polynomial mapping
Gross Production (scaled)
Time (sampling interval 30 seconds)
0
1
2
3
4
5
6
7
8
0
500
1000
1500
2000
2500
3000

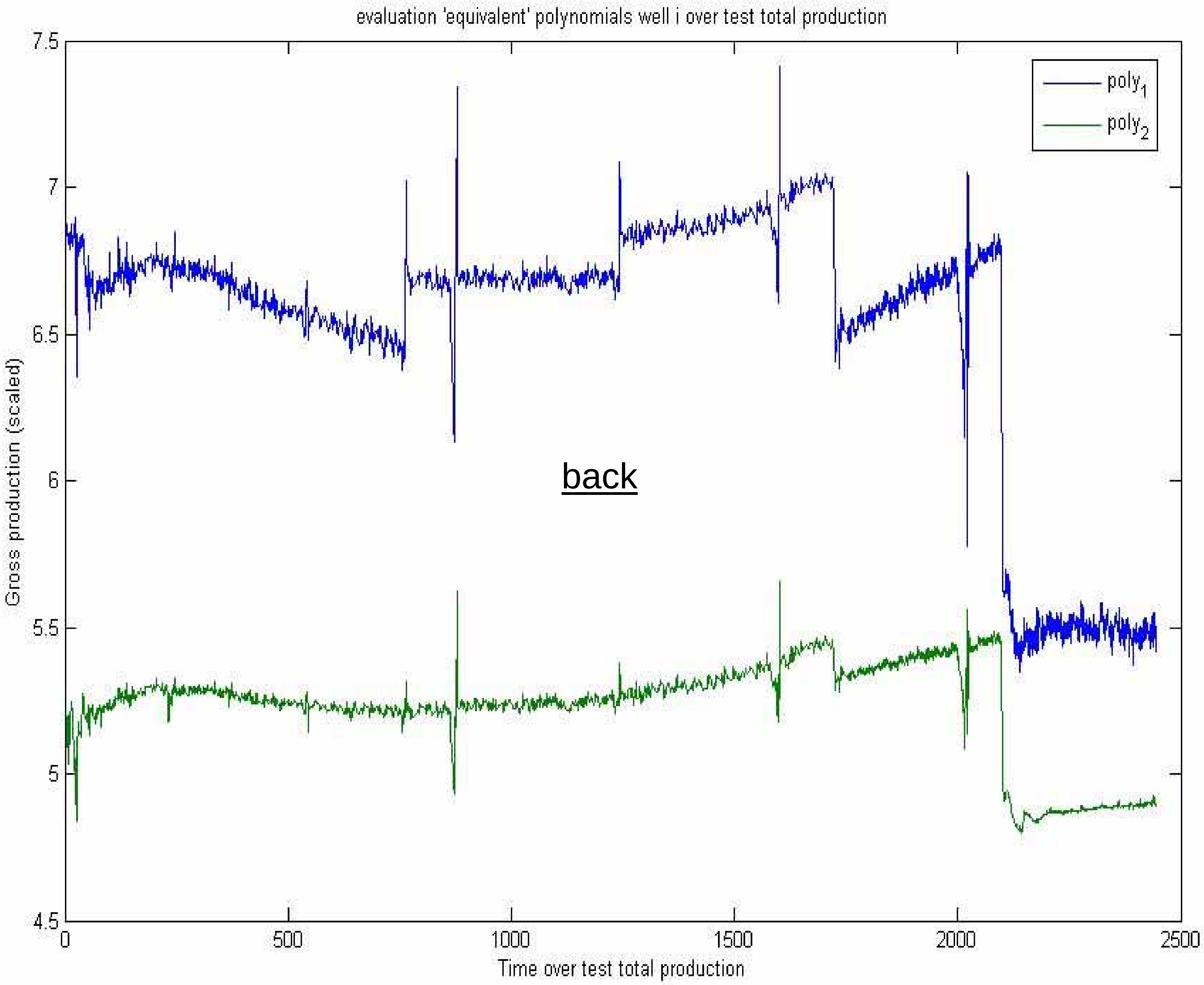
Back


empirical polynomial total production
measured total production
polynomial mapping
Gross Production (scaled)
Time (Sampling Interval 30 seconds)
13
12
11
10
9
8
7
6
5
0
500
1000
1500
2000
2500

Back


different polynomials f_1, 'equivalent' evaluations
Gross production (scaled)
Time over individual production well 1
0
1
2
3
4
5
6
7
8
0
500
1000
1500
2000
2500
3000
measurements
$poly_1$
$poly_2$
$poly_3$
$poly_4$

Back


evaluation 'equivalent' polynomials well i over test total production
Gross production (scaled)
Time over test total production
7.5
7
6.5
6
5.5
5
4.5
0
500
1000
1500
2000
2500
poly_1
poly_2

back