

Contents

1	Endomorphisms	1
1.1	Generalized Eigenspaces	2
1.1.A	Big Kernels and Small Images	3
1.1.B	Minimal Polynomials and Eigenspaces	6
1.2	Minimal and Characteristic Polynomials	12
1.3	Nilpotent Endomorphisms and Multiplicities	17
1.4	The Module Structure Given by an Endomorphism	24
1.5	Commendable Endomorphisms	34
1.6	Other Special Endomorphisms	40
2	Families of Commuting Endomorphisms	47
2.1	Commuting Families	49
2.2	Kernels and Big Kernels of Ideals	54
2.2.A	Properties of Zero-Dimensional Rings	55
2.2.B	Properties of Kernels and Big Kernels of Ideals	59
2.3	Eigenfactors	64
2.4	Joint Eigenspaces	69
2.5	Splitting Endomorphisms	77
2.6	Simultaneous Diagonalization and Triangularization	89
3	Special Families of Endomorphisms	95
3.1	\mathcal{F} -Cyclic Vector Spaces	96
3.2	Unigenerated Families	101
3.3	Commendable Families	110
3.4	Local Families	113
3.5	Dual Families	119
3.6	Extended Families	126
4	Zero-Dimensional Affine Algebras	131
4.1	Multiplication Endomorphisms	134
4.2	Primary Decomposition and Separators	139
4.3	Commendable and Splitting Multiplication Endomorphisms	145

4.4	Local Multiplication Families	153
4.5	Dual Multiplication Families	161
4.6	Hilbert Functions and the Cayley-Bacharach Property	175
5	Computing Primary and Maximal Components	185
5.1	Computing Primary Decompositions	188
5.1.A	Using the Generically Extended Linear Form	189
5.1.B	Using Linear Forms and Idempotents	194
5.1.C	Computing Joint Eigenspaces	200
5.2	Primary Decomposition over Finite Fields	203
5.3	Computing Maximal Components via Factorization	213
5.3.A	Minimal Polynomials and Finite Field Extensions	213
5.3.B	Factorizing over Extension Fields	218
5.3.C	Using Factorizations to Compute Maximal Components	223
5.4	Primary Decompositions Using Radical Ideals	226
5.4.A	Maximal Components via Radical Ideals	227
5.4.B	Primary Components from Maximal Components	233
5.5	The Separable Subalgebra	237
6	Solving Zero-Dimensional Polynomial Systems	243
6.1	Rational Zeros via Commuting Families	245
6.1.A	Computing One-Dimensional Joint Eigenspaces	246
6.1.B	Computing Linear Maximal Ideals	252
6.2	Rational Zeros via Eigenvalues and Eigenvectors	255
6.2.A	The Eigenvalue Method	256
6.2.B	The Eigenvector Method	264
6.3	Solving Polynomial Systems over Finite Fields	269
6.3.A	Computing Isomorphisms of Finite Fields	271
6.3.B	Solving over Finite Fields via Cloning	279
6.3.C	Solving over Finite Fields via Univariate Representations	285
6.3.D	Solving over Finite Fields via Recursion	290
6.4	Solving Polynomial Systems over the Rationals	295
6.4.A	Splitting Fields in Characteristic Zero	297
6.4.B	Solving over the Rational Numbers via Cloning	302
	Notation	311
	References	315
	Index	317