Discussion on the paper by Gerber and Chopin

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I congratulate Gerber and Chopin for this extraordinary contribution and I am very happy that now quasi-Monte-Carlo approaches will achieve wider recognition also in statistics. The numerical experiments in the paper are very promising and I believe that the theoretic results are not yet as far developed as the experiments indicate. I would like to comment on the consistency result of Section 3.4. For me the main challenge would be to derive an explicit error bound instead of the asymptotic statement from theorem 5.

To achieve such a result an explicit estimate of the importance sampling procedure (see theorem 1 in Section 3.2) might be useful. As the authors already point out, results in that direction can be found in [1].

However, this is certainly not enough to obtain an explicit error estimate. Since this is a difficult question we might start with asking whether for all $T \in \mathbb{N}$ and all $N \in \mathbb{N}$ there is a sequence of deterministic point sets $\mathcal{U}_{T,N} = (U_{t,N})_{0 \leq t \leq T}$ with

$$U_{t,N} = \{u_t^1, \dots, u_t^N\} \subset [0,1)^{d+1},$$

such that the approximation scheme $\widehat{\mathbb{Q}}_t^N$, given by algorithm 3, using $\mathcal{U}_{T,N}$ satisfies

$$\|\widehat{\mathbb{Q}}_t^N - \mathbb{Q}_t\|_E \leq r(N), \qquad \forall t \in \{0, 1, \dots, T\},$$

with an explicit known function $r \colon \mathbb{N} \to \mathbb{R}_+$ which goes to zero as N increases. For example, one such function could be $r(N) = C_1/\sqrt{N}$ where the constant C_1 might even be independent of the dimension. Or, as for quasi-Monte-Carlo approaches very common, $r(N) = C_2/N^{1-\varepsilon}$ for any $\varepsilon > 0$, but here C_2 scales probably not very well with respect to the dimension.

Let me finally mention that such types of question are also asked in other contexts and can sometimes be answered with probabilistic arguments; see for example [3]. This strategy is also related to the considerations in [2] and [4].

References

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